

Chapter 3 roadmap:

- ▶ Propositions, boolean logic, logical equivalences. **Game 1** (last week Monday)
- ▶ Conditional propositions. **SML** (last week Wednesday)
- ▶ Arguments. **Game 2** (last week Friday)
- ▶ Predicates and quantification. **SML** (Monday)
- ▶ Quantified arguments. **Game 3** (Today)
- ▶ Review for test. (Friday)
- ▶ Test 1. (next week Monday)

Today:

- ▶ Finish programming examples from last time
- ▶ Think through quantified argument forms
- ▶ Practice verifying validity of quantified argument forms (Game 3)

Project proposal due Wednesday, Feb 14.

Common forms for propositions

$$\forall x \in A, P(x)$$

$$\forall x \in A, P(x) \rightarrow Q(x)$$

$$\exists x \in A \mid P(x)$$

Universal instantiation

$\forall x \in A, P(x)$
 $a \in A$
 $\therefore P(a)$

Universal modus tollens

$\forall x \in A, P(x) \rightarrow Q(x)$
 $a \in A$
 $\sim Q(a)$
 $\therefore \sim P(a)$

Existential instantiation

$\exists x \in A \mid P(x)$
Let $a \in A \mid P(a)$
 $\therefore a \in A \wedge P(a)$

Universal modus ponens

$\forall x \in A, P(x) \rightarrow Q(x)$
 $a \in A$
 $P(a)$
 $\therefore Q(a)$

Existential Generalization

$a \in A$
 $P(a)$
 $\therefore \exists x \in A \mid P(x)$

Hypothetical conditional

Suppose p
 q
 $\therefore p \rightarrow q$

Universal generalization

Suppose $a \in A$
 $P(a)$
 $\therefore \forall x \in A, P(x)$

Hypothetical division into cases

$p \vee q$
Suppose p
 r
Suppose q
 r
 $\therefore r$

3.14.4

- a. $\forall x \in A, P(x) \wedge \sim Q(x)$
- b. $\forall x \in A, x \in B$
- c. $\forall x \in B, \sim Q(x) \rightarrow R(x)$
- d. $\therefore \forall x \in A, R(x)$

3.14.5

a. $\forall x \in A, x \in B$

b. $\forall x \in B, \sim P(x)$

c. $\forall x \in A, Q(x) \rightarrow P(x)$

d. $\therefore \forall x \in A, \sim Q(x)$

(Extra # 1)

(a) $\forall y \in B, \exists x \in A \mid R(x, y)$

(b) $\forall x \in A, \forall y \in B, (P(x) \wedge R(x, y) \rightarrow Q(y))$

(c) $\therefore (\forall x \in A, P(x)) \rightarrow (\forall y \in B, Q(y))$

(Extra # 2)

(a) $\forall x \in A, P(x)$

(b) $\forall x \in A, x \in B \vee R(x)$

(c) $\forall y \in B, Q(y) \vee \sim P(y)$

(d) $\forall x \in A, R(x) \rightarrow Q(x)$

(e) $\therefore \forall x \in A, Q(x)$

(Extra # 3)

(a) $\forall x \in A, P(x) \rightarrow R(x)$

(b) $\exists x \in A \mid P(x)$

(c) $\forall x \in A, Q(x) \vee x \in B$

(d) $\forall x \in A, P(x) \rightarrow \sim Q(x)$

(e) $\therefore \exists y \in B \mid R(y)$

3.14.10

- a. $\forall x \in A, \exists y \in B \mid P(x, y)$
- b. $\forall y \in B, Q(y) \vee R(y)$
- c. $\forall x \in A, y \in B, P(x, y) \rightarrow \sim Q(y)$
- d. $\exists x \in A \mid S(x)$
- e. $\therefore \exists y \in B \mid R(y)$

3.14.11

- a. $\forall x \in A, x \in B \wedge x \in C$
- b. $\forall x \in C, x \in D \vee x \in E$
- c. $\forall x \in B, x \in D \rightarrow P(x)$
- d. $\forall x \in B, x \in E \rightarrow Q(x)$
- e. $\forall x \in B, P(x) \vee Q(x) \rightarrow R(x)$
- f. $\therefore \forall x \in A, R(x)$

For next time:

Pg 139: 3.14.(6-9)

See Canvas for fully-parenthesized versions of Game 3 problems.