

## Chapter 3 roadmap:

- ▶ Propositions, boolean logic, logical equivalences. **Game 1** (last week Monday)
- ▶ Conditional propositions. **SML** (last week Wednesday)
- ▶ Arguments. **Game 2** (last week Friday)
- ▶ Predicates and quantification. **SML** (Monday)
- ▶ Quantified arguments. **Game 3** (Wednesday)
- ▶ Review for test. (Today)
- ▶ Test 1 (next week Monday)
- ▶ Begin Chapter 4, Proofs (next week Wednesday)

## Today:

- ▶ General comments on tests in this course
- ▶ Broad overview of everything so far
- ▶ Test 1 specifics
- ▶ Warnings and clarifications

Project proposal due Wednesday, Feb 14.

Which of the following are true?

$$-\left((x - y) + (x - z)\right) \equiv -(x - y) - (x - z)$$

$$-\left((x - y) + (x - z)\right) \cdot z \equiv -(x - y) - (x - z) \cdot z$$

$$\sim (p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim (p \wedge q) \wedge r \equiv \sim p \vee \sim q \wedge r$$

Which of the following are true?

$$(x + y) + z = x + (y + z)$$

$$(x - y) + z = x - (y + z)$$

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

$$(p \vee q) \wedge r \equiv p \vee (q \wedge r)$$

1. Write a function `leastSigDigs` that takes a list of ints and returns a list of the least significant digits in those lists. For example, `leastSigDigs[283, 7234, 5, 2380]` would return `[3, 4, 5, 0]`.

2. Write a function `hasEmpty` that takes a list of lists (of any type) and determines whether or not the list of lists contains an empty list. For example, `hasEmpty([[1,2,3], [4,5], [], [6,7]])` would return `true`.

### Universal instantiation

$\forall x \in A, P(x)$   
 $a \in A$   
 $\therefore P(a)$

### Universal modus tollens

$\forall x \in A, P(x) \rightarrow Q(x)$   
 $a \in A$   
 $\sim Q(a)$   
 $\therefore \sim P(a)$

### Existential instantiation

$\exists x \in A \mid P(x)$   
Let  $a \in A \mid P(a)$   
 $\therefore a \in A \wedge P(a)$

### Universal modus ponens

$\forall x \in A, P(x) \rightarrow Q(x)$   
 $a \in A$   
 $P(a)$   
 $\therefore Q(a)$

### Existential Generalization

$a \in A$   
 $P(a)$   
 $\therefore \exists x \in A \mid P(x)$

### Hypothetical conditional

Suppose  $p$   
 $q$   
 $\therefore p \rightarrow q$

### Universal generalization

Suppose  $a \in A$   
 $P(a)$   
 $\therefore \forall x \in A, P(x)$

### Hypothetical division into cases

$p \vee q$   
Suppose  $p$   
 $r$   
Suppose  $q$   
 $r$   
 $\therefore r$

(Extra # 2)

(a)  $\forall x \in A, P(x)$

(b)  $\forall x \in A, x \in B \vee R(x)$

(c)  $\forall y \in B, Q(y) \vee \sim P(y)$

(d)  $\forall x \in A, R(x) \rightarrow Q(x)$

(e)  $\therefore \forall x \in A, Q(x)$

Suppose  $a \in A$

(i)  $a \in B \wedge R(a)$

Suppose  $a \in B$

(ii)  $Q(a) \vee \sim P(a)$

(iii)  $P(a)$

(iv)  $Q(a)$

Suppose  $R(a)$

(v)  $Q(a)$

(vi)  $Q(a)$

(vii)  $\therefore \forall x \in A, Q(x)$

by supposition, (b), and UI

by supposition, (c), and UI

by supposition, (a), and UI

by (ii), (iii), and elimination

by supposition, (c), and UMP

by (i), (iv),(v), and HDC

by supposition, (vi), and UG

(Extra # 3)

(a)  $\forall x \in A, P(x) \rightarrow R(x)$

(b)  $\exists x \in A \mid P(x)$

(c)  $\forall x \in A, Q(x) \vee x \in B$

(d)  $\forall x \in A, P(x) \rightarrow \sim Q(x)$

(e)  $\therefore \exists y \in B \mid R(y)$

Let  $a \in A \mid P(a)$

(i)  $a \in A \wedge P(a)$

(ii)  $a \in A$

(iii)  $P(a)$

(iv)  $\sim Q(a)$

(v)  $Q(a) \vee a \in B$

(vi)  $a \in B$

(vii)  $R(a)$

(viii)  $\therefore \exists y \in B \mid R(y)$

By (b) and EI

By (i) and specialization

By (i) and specialization

by (ii), (iii), (d), and UMP

by (ii), (c), and UI

by (iv), (v), and elimination

by (ii), (iii), (a), and UMP

by (vi), (vii), and EG