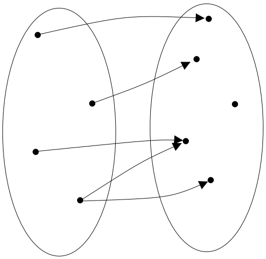
Chapter 7 outline:

- Introduction, function equality, and anonymous functions (last week Friday)
- Image and inverse images (**Today**)
- Function properties, composition, and applications to programming (Wednesday)
- Cardinality (Friday)
- Countability (next week Monday)
- Review (next week Wednesday)
- Test 3, on Ch 6 & 7 (next week Friday)

Today:

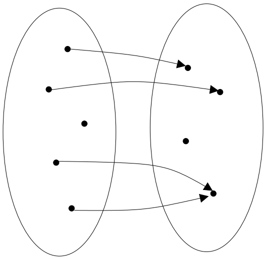
- Review definitions from last time
- New definitions: image and inverse image
- Proofs
- Programming

A relation f from X to Y is a function (written $f : X \to Y$) if $\forall\ x \in X$,
(1) $\exists\ y \in Y \mid (x, y) \in f$, and (2) $\forall\ y_1, y_2 \in Y$, $(x, y_1), (x, y_2) \in f \to y_1 = y_2$.



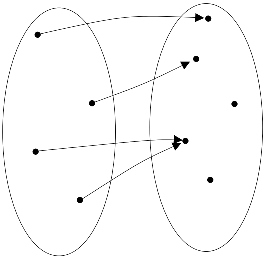
Not a function.

(There's a domain element that is related to two things.)



Not a function.

(There's a domain element that is not related to anything.)

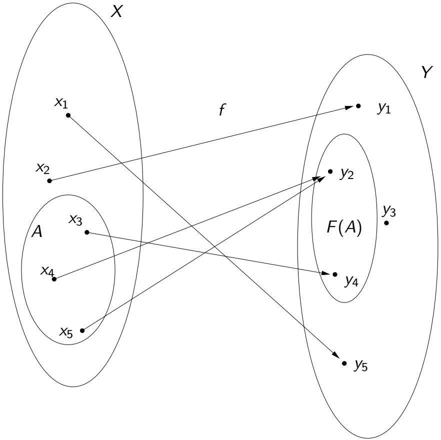


A function.

(It's OK that two domain elements are related to the same thing and one codomain element has nothing related to it.)

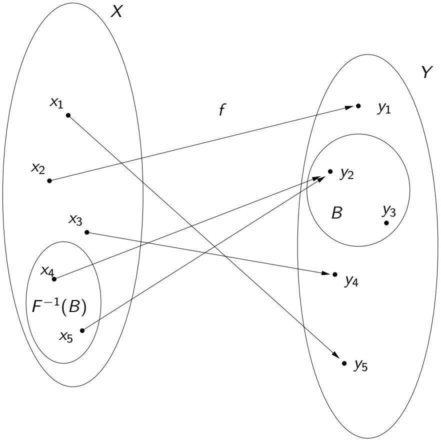
Image

$$F(A) = \{y \in Y \mid \exists\, x \in A \text{ such that } f(x) = y\}$$



Inverse image

$$F^{-1}(B) = \{x \in X \mid f(x) \in B\}$$



Lemma 7.2. If $f : X \to Y$, then $F(\emptyset) = \emptyset$.

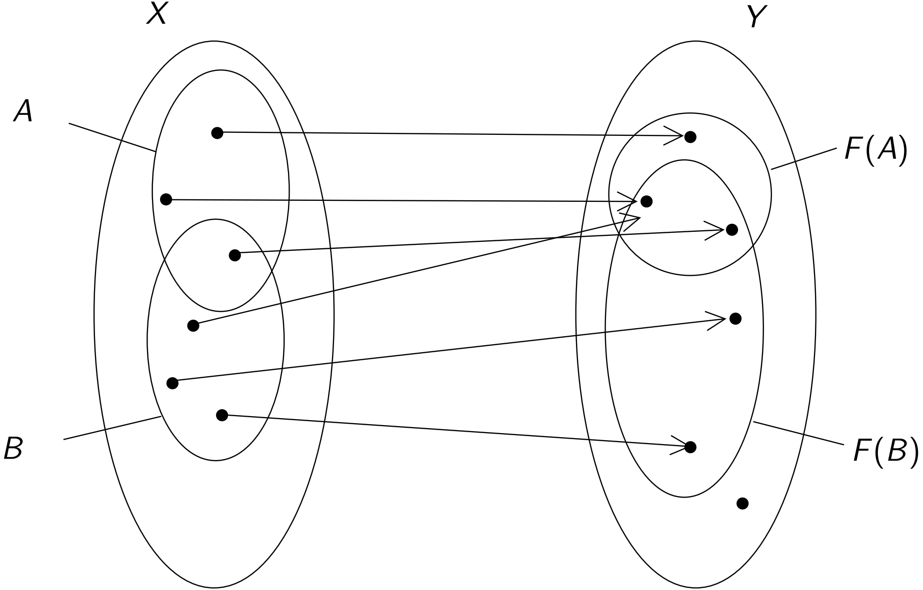
Lemma 7.3. If $f : X \to Y$, $A \subseteq X$, and $A \neq \emptyset$, then $F(A) \neq \emptyset$.

Lemma 7.4. If $f : X \to Y$, then $F^{-1}(\emptyset) = \emptyset$.

We might expect the following, but *it's not true*:

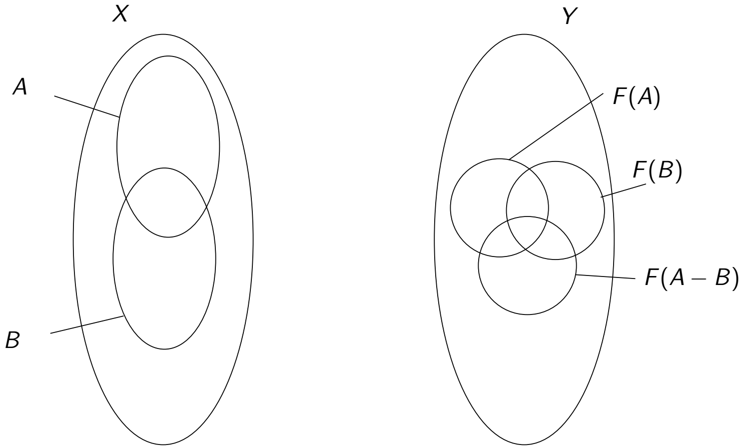
Lemma XXXX. If $f : X \to Y$, $A \subseteq Y$, and $A \neq \emptyset$, then $F^{-1}(A) \neq \emptyset$.

Ex 7.4.1. If $A, B \subseteq X$, then $F(A \cap B) \subseteq F(A) \cap F(B)$.



Ex 7.4.3. If $A, B \subseteq X$, then $F(A - B) \subseteq F(A) - F(B)$?

Consider this picture of X and Y:



Ex 7.4.3. If $A, B \subseteq X$, then $F(A - B) \subseteq F(A) - F(B)$?

Attempted proof. Suppose $A, B \subseteq X$ and $y \in F(A - B)$. By definition of image, there exists $x \in A - B$ such that $f(x) = y$.

Ex 7.4.3. If $A, B \subseteq X$, then $F(A - B) \subseteq F(A) - F(B)$?

Attempted proof. Suppose $A, B \subseteq X$ and $y \in F(A - B)$. By definition of image, there exists $x \in A - B$ such that $f(x) = y$.

By definition of difference, $x \in A$, and $x \notin B$. By definition of image, $f(x) \in F(A)$.

Ex 7.4.3. If $A, B \subseteq X$, then $F(A - B) \subseteq F(A) - F(B)$?

Attempted proof. Suppose $A, B \subseteq X$ and $y \in F(A - B)$. By definition of image, there exists $x \in A - B$ such that $f(x) = y$.

By definition of difference, $x \in A$, and $x \notin B$. By definition of image, $f(x) \in F(A)$.

So, also by definition of image, $f(x) \notin F(B)$. Right?

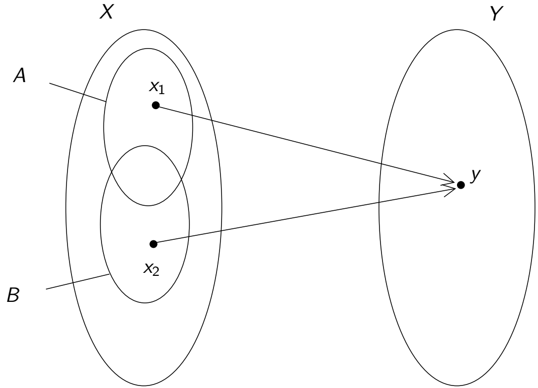
Ex 7.4.3. If $A, B \subseteq X$, then $F(A - B) \subseteq F(A) - F(B)$?

Attempted proof. Suppose $A, B \subseteq X$ and $y \in F(A - B)$. By definition of image, there exists $x \in A - B$ such that $f(x) = y$.

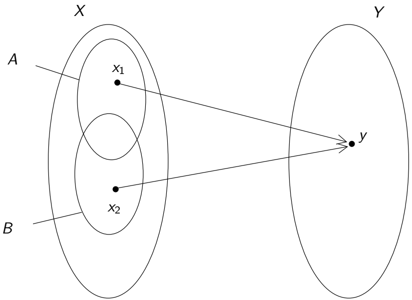
By definition of difference, $x \in A$, and $x \notin B$. By definition of image, $f(x) \in F(A)$.

So, also by definition of image, $f(x) \notin F(B)$. Right?

NO!



Ex 7.4.3. If $A, B \subseteq X$, then $F(A - B) \subseteq F(A) - F(B)$?



Let $X = \{x_1, x_2\}$, $Y = \{y\}$, $A = \{x_1\}$, and $B = \{x_2\}$.

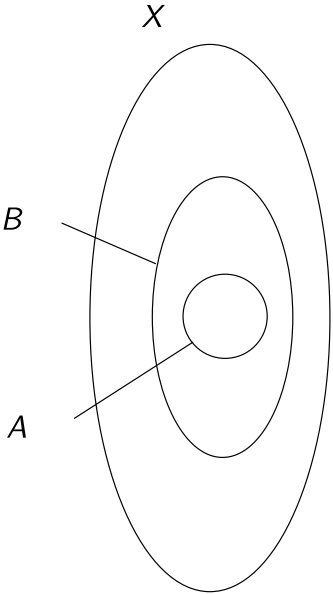
Let $f = \{(x_1, y), (x_2, y)\}$.

Then $F(A - B) = F(\{x_1\} - \{x_2\}) = F(\{x_1\}) = \{y\}$.

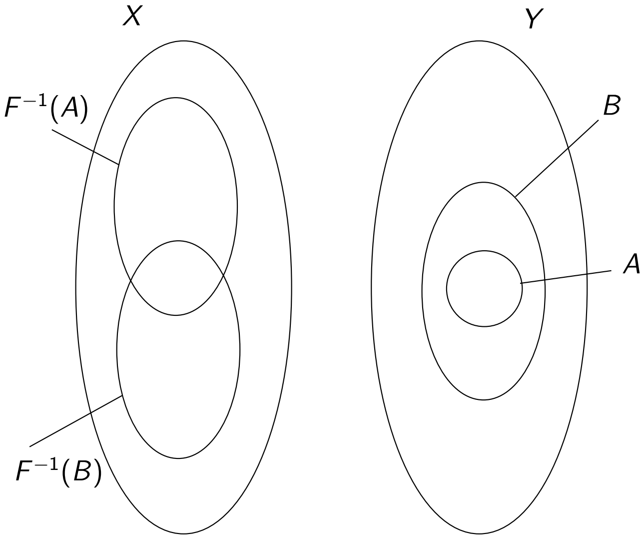
Moreover, $F(A) - F(B) = \{y\} - \{y\} = \emptyset$.

So $F(A - B) \not\subseteq F(A) - F(B)$

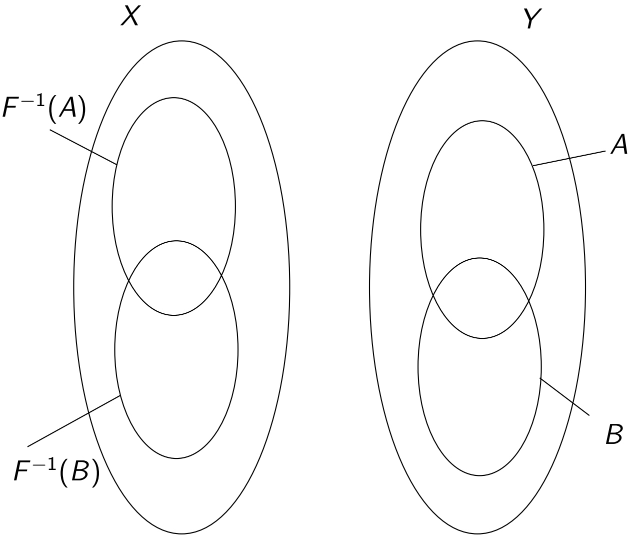
Ex 7.4.4. If $A \subseteq B \subseteq X$, then $F(B) = F(B - A) \cup F(A)$.



Ex 7.4.6. If $A \subseteq B \subseteq Y$, then $F^{-1}(A) \subseteq F^{-1}(B)$.



Ex 7.4.7. If $A, B \subseteq Y$, then $F^{-1}(A \cup B) = F^{-1}(A) \cup F^{-1}(B)$.



For next time:

Pg 342: 7.4.(2, 5, 8, 9, 10)
(Programming problems are with the next assignment)

Read 7.(6-8)
Take quiz