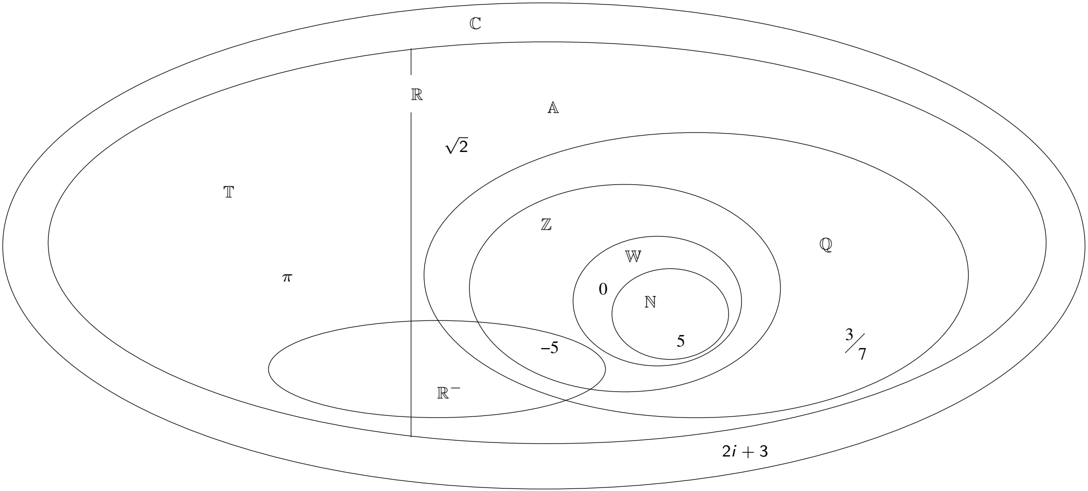
Chapter 1 outline:

- Introduction, sets and elements (this past Monnday)
- Set operations; visual verification of set propositions (**Today**)
- Introduction to SML; cardinality and Cartesian products (Friday)
- Making types in SML (next week Wednesday)
- Making functions in SML (next week Friday)

Today:

- Set symbols and terminology
- Set notation
- Set operations
- Verifying set equivalence visually



$\mathbb{C}$
$\mathbb{R}$
$\mathbb{A}$
$\sqrt{2}$
$\mathbb{T}$
$\mathbb{Z}$
$\mathbb{Q}$
$\mathbb{W}$
π
0
$\mathbb{N}$
5
-5
$3/7$
$\mathbb{R}^-$
$2i + 3$

5 is a natural number; *or* the collection of natural numbers contains 5. $5 \in \mathbb{N}$

Adding 0 to the collection of natural numbers makes the collection of whole numbers. $\mathbb{N} \cup \{0\} = \mathbb{W}$

Merging the algebraic numbers and the transcendental numbers makes the real numbers. $\mathbb{A} \cup \mathbb{T} = \mathbb{R}$

Transcendental numbers are those real numbers which are not algebraic numbers. $\mathbb{T} = \mathbb{R} - \mathbb{A}$

Nothing is both transcendental and algebraic, *or* the collection of things both transcendental and algebraic is empty. $\mathbb{T} \cap \mathbb{A} = \emptyset$

Negative integers are both negative and integers. $\mathbb{Z}^- = \mathbb{Z} \cap \mathbb{R}^-$

All integers are rational numbers. $\mathbb{Z} \in \mathbb{R}$

Since all rational numbers are algebraic numbers and all algebraic numbers are real numbers, it follows that all rational numbers are real numbers.

$$\begin{aligned} & \mathbb{Q} \subseteq \mathbb{A} \\ & \mathbb{A} \subseteq \mathbb{R} \\ \therefore \quad & \mathbb{Q} \subseteq \mathbb{R} \end{aligned}$$

Axiom (Existence.)

There is a set with no elements.

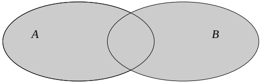
Axiom (Extensionality.)

If every element of a set X is an element of a set Y and every element of Y is an element of X, then $X = Y$.

Union

$A \cup B = \{\ x \mid x \in A \text{ or } x \in B\}$

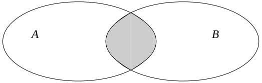
$$\begin{aligned} \{1,2,3\} \cup \{2,3,4\} &= \{1,2,3,4\} \\ \{1,2\} \cup \{3,4\} &= \{1,2,3,4\} \\ \{1,2\} \cup \{1,2,3\} &= \{1,2,3\} \end{aligned}$$



Intersection

$A \cap B = \{\ x \mid x \in A \text{ and } x \in B\}$

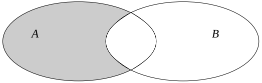
$$\begin{aligned} \{1,2,3\} \cap \{2,3,4\} &= \{2,3\} \\ \{1,2\} \cap \{3,4\} &= \emptyset \\ \{1,2\} \cap \{1,2,3\} &= \{1,2\} \end{aligned}$$



Difference

$A - B = \{\ x \mid x \in A \text{ and } x \notin B\}$

$$\begin{aligned} \{1,2,3\} - \{2,3,4\} &= \{1\} \\ \{1,2\} - \{3,4\} &= \{1,2\} \\ \{1,2\} - \{1,2,3\} &= \emptyset \end{aligned}$$



1. $\{1,2,3,4,5\} \cup \{5,6,7\} =$

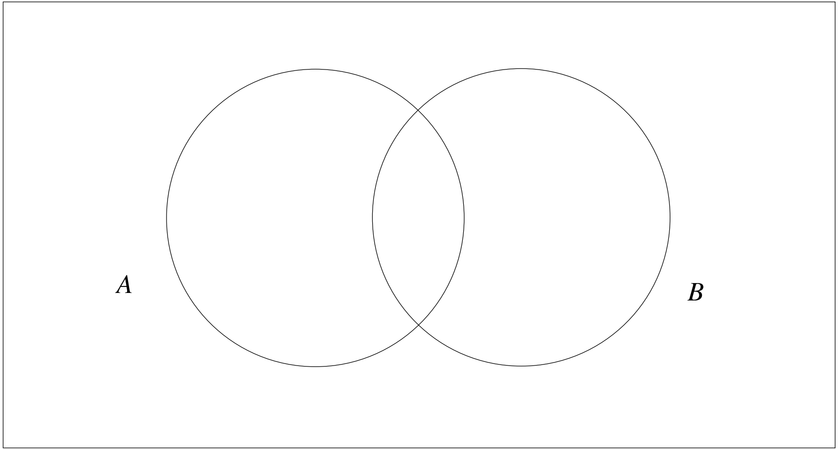
2. $\{1,2,3,4,5\} \cap \{2,4,6,8,10\} =$

3. $\{1,2,3,4,5\} - \{2,3,4\} =$

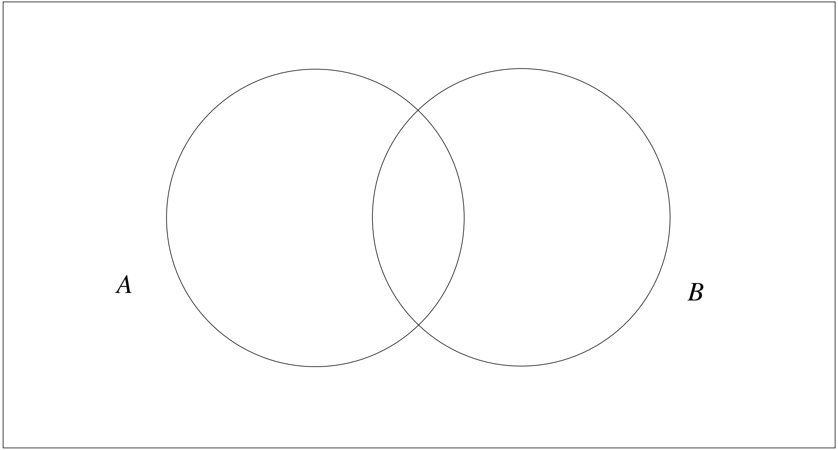
4. $\{1,2,3,4,5\} - \{3,4,5,6,7\} =$

Which of the following are equal to $\{1, 2, 3, 4\}$?

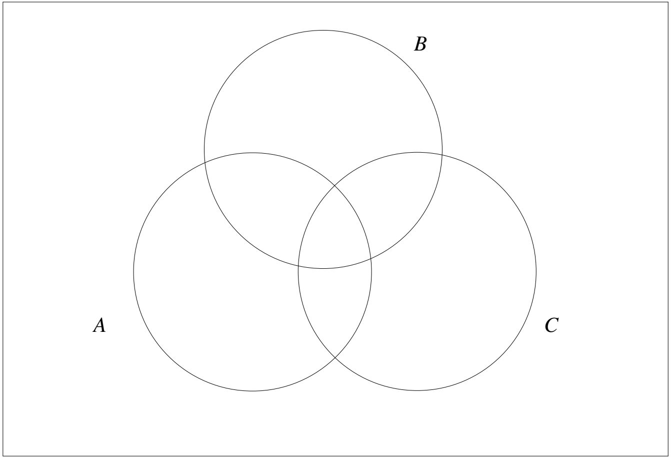
- $\{1, 2\} \cup \{3, 4\}$
- $\{1, 2, 3\} \cup \{4\}$
- $\{1, 2, 3\} \cup \{2, 3, 4\}$
- $\{1, 2, 3\} \cup \{3, 4, 5\}$
- $\{2, 3\} \cup \{1, 4\}$
- $\{1\} \cup \{3, 4\}$
- $\{4, 3, 2, 1\}$
- $\{1\} \cup \{1, 2\} \cup \{1, 2, 3\} \cup \{1, 2, 3, 4\}$



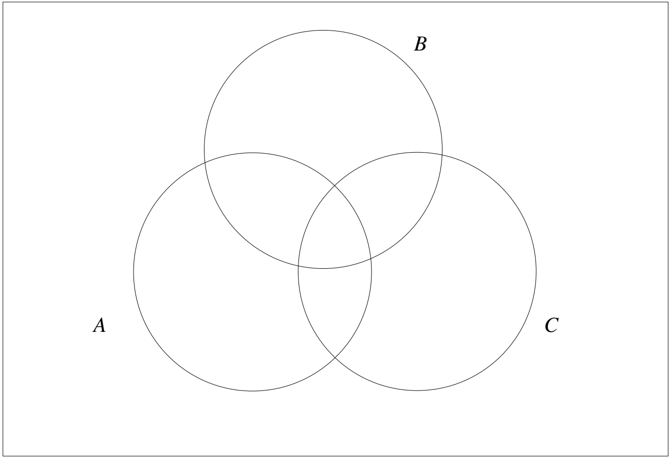
1.4.7. $(A \cap B) - A$



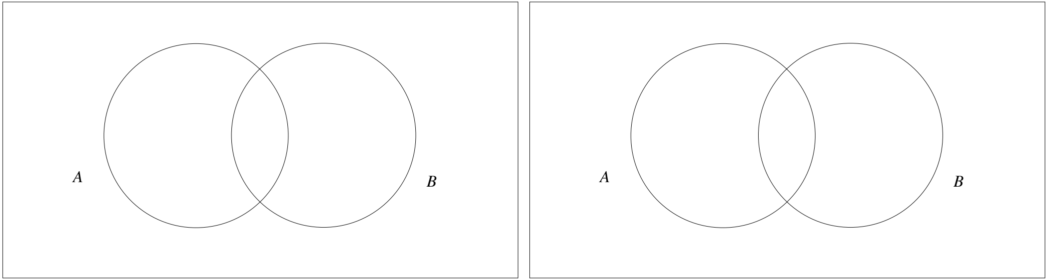
1.4.8. $(A - B) \cup (B - A)$



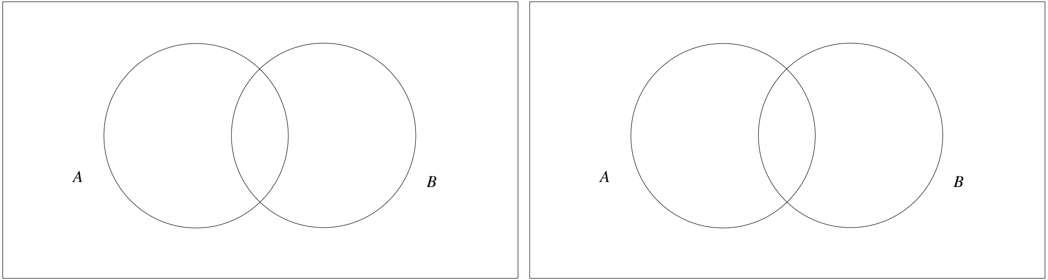
1.4.9. $(A \cup B) \cap (A \cup C)$



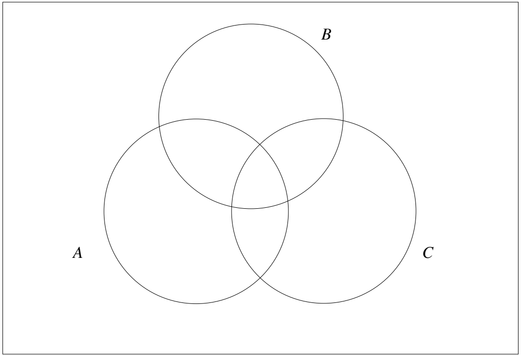
1.4.10. $\overline{(A \cap B)} \cap (A \cup C)$

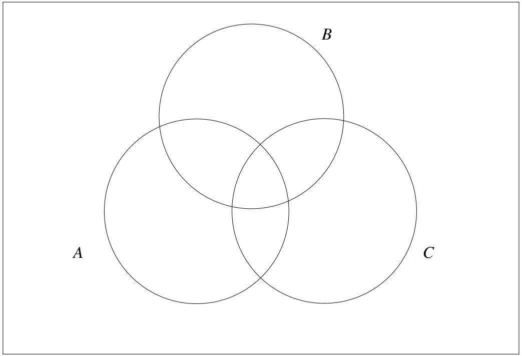


$$A \cup (A \cap B) = A$$

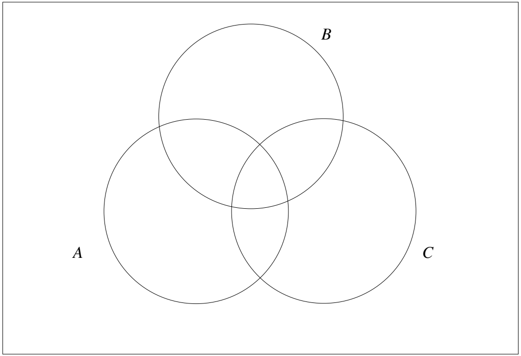


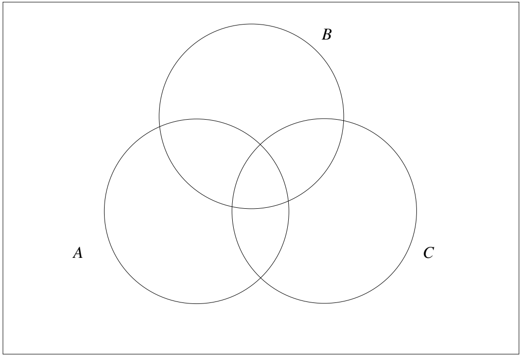
$$A \cup \overline{A} = \mathcal{U}$$



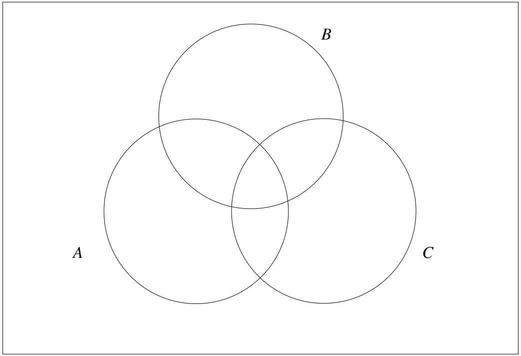


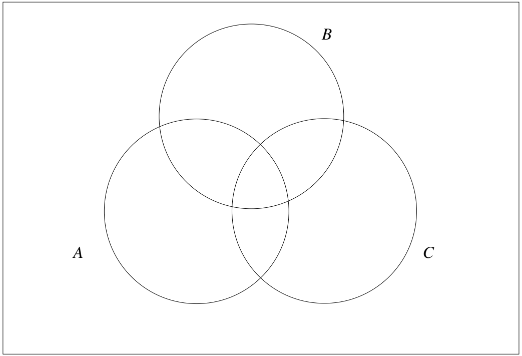
$$A \cup (B \cup C) = (A \cup B) \cup C$$



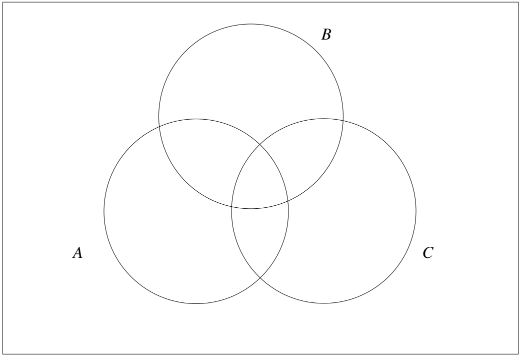


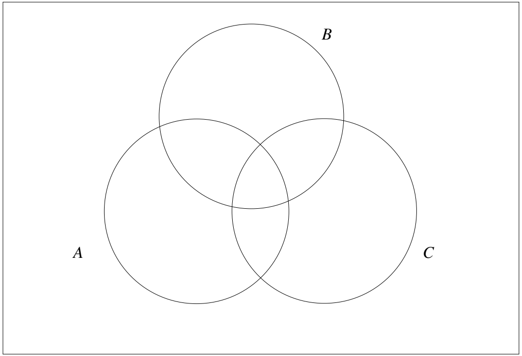
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



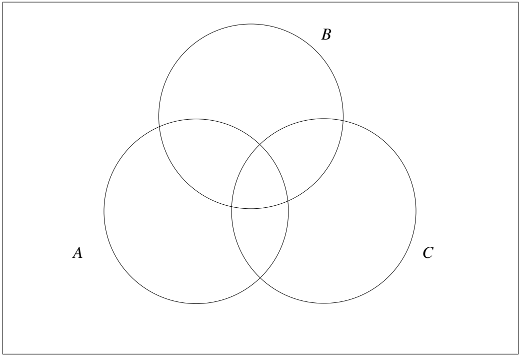


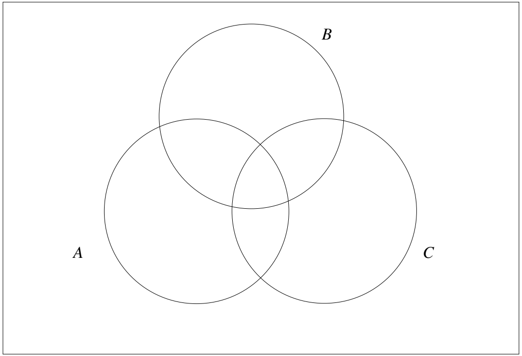
$$A \cap B = A - (A - B)$$



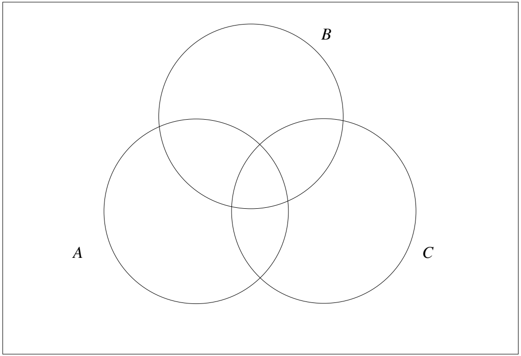


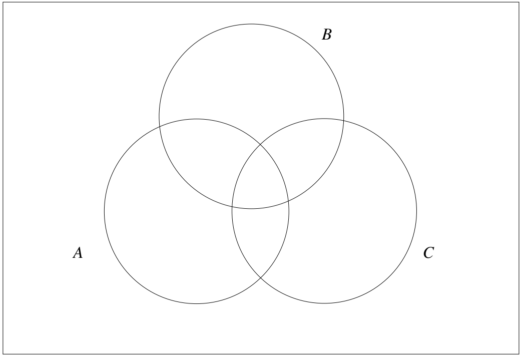
$$(A \cap C) - (C - B) = A \cap B \cap C$$





$$A \cup (A - B) = A$$





$$(A \cup (B - C)) \cap \overline{B} = A - B$$

For next time:

Pg 12: 1.3.(11-14, 16)
Pg 16: 1.4.(1-6, 19)
Pg 20: 1.5.(8-11)

Read 1.(6-9)

Take quiz