Chapter 4 roadmap:

- Subset proofs (last week Friday)
- Set equality and emptiness proofs (Monday)
- Conditional and biconditional proofs (**Today**)
- Proofs about powersets (Friday)
- Review for Test 2 (next week Monday)
- Test 2, on Chapters 3 & 4 (next week Wednesday, Mar 5)

Today:

- Proofs of conditional propositions
- Proofs about numbers
- Proofs of biconditional propositions

General forms:

1. Facts ($p$)
   Set forms
   1. Subset $X \subseteq Y$
   2. Set equality $X = Y$
   3. Set emptiness $X = \emptyset$
2. Conditionals ($p \to q$)
3. Biconditionals ($p \leftrightarrow q$)

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $T$ |
| $F$ | $F$ | $T$ |

To prove $p \to q$

Suppose $p$

$\ldots$

$q$

$p \to q$

An integer $x$ is *even* if $\exists\ k \in \mathbb{Z} \mid x = 2k$.

An integer $x$ is *odd* if $\exists\ k \in \mathbb{Z} \mid x = 2k + 1$.

"Axiom 3." If $x, y \in \mathbb{Z}$, then $x + y \in \mathbb{Z}$. (*Closure of addition*)

"Axiom 4." If $x, y \in \mathbb{Z}$, then $x \cdot y \in \mathbb{Z}$. (*Closure of multiplication*)

"Axiom 5." If $x \in \mathbb{Z}$, then $x$ is even iff $x$ is not odd.

$\forall\ x, y \in \mathbb{Z}$, $x \mid y$ (read, "$x$ divides $y$") if $\exists\ k \in \mathbb{Z} \mid x \cdot k = y$.

Note that $y/x = k$ or $\frac{y}{x} = k$ or $x\overset{\displaystyle k}{\overline{\big)\ y}}$.

**For next time:**

*Pg 162: 4.4.(1, 4, 5)*
*Pg 164: 4.5.(2 & 5)*

*Review 1.8, especially Ex 1.8.14*
*Skim 4.7*

*Take quiz*