

Chapter 4 roadmap:

- ▶ Subset proofs (last week Friday)
- ▶ Set equality and emptiness proofs (**Today**)
- ▶ Conditional and biconditional proofs (Wednesday)
- ▶ Proofs about powersets (Friday)
- ▶ (Test 2, Wednesday, Mar 5)
- ▶ (Begin Ch 5, Relations, next week Fri)

Today:

- ▶ Proofs that sets are equal
- ▶ Proofs that a set is empty

General forms:

1. Facts (p) Set forms

1. Subset $X \subseteq Y$
2. Set equality $X = Y$
3. Set emptiness $X = \emptyset$

2. Conditionals ($p \rightarrow q$)

3. Biconditionals ($p \leftrightarrow q$)

$$A \times (B - C) \subseteq (A \times B) - (A \times C).$$

Proof (long version). Suppose $x \in A \times (B - C)$. By definition of Cartesian product, $x = (a, d)$ for some $a \in A$ and $d \in B - C$. By definition of difference, $d \in B$ and $d \notin C$.

By definition of Cartesian product, $(a, d) \in A \times B$. Also by definition of Cartesian product, this time used negatively, $(a, d) \notin A \times C$.

[That is, we rewrite $d \notin C$. as $\sim (d \in C)$. By generalization, $\sim (d \in C \wedge a \in A)$. By definition of Cartesian product, $\sim ((a, d) \in A \times C)$. This can be rewritten as $(a, d) \notin A \times C$.]

By definition of difference, $(a, d) \in (A \times B) - (A \times C)$. By substitution, $x \in (A \times B) - (A \times C)$. Therefore, by definition of subset, $A \times (B - C) \subseteq (A \times B) - (A \times C)$. \square

$$A \times (B - C) \subseteq (A \times B) - (A \times C).$$

Proof (short version). Suppose $(a, d) \in A \times (B - C)$. By definition of Cartesian product, $a \in A$ and $d \in B - C$.

By definition of difference, $d \in B$ and $d \notin C$. By definition of Cartesian product, $(a, d) \in A \times B$ and $(a, d) \notin A \times C$.

By definition of difference, $(a, d) \in (A \times B) - (A \times C)$. Therefore, by definition of subset, $A \times (B - C) \subseteq (A \times B) - (A \times C)$. \square

For next time:

Pg 160: 4.2.(3, 14, 15, 18)

Pg 161: 4.3.(5 & 6)

See assignment on Canvas for hint on Ex 4.2.15.

Read 4.(4 & 5)

Take quiz