Chapter 5 roadmap:

- Introduction to relations (spring-break eve)
- Properties of relations (last week Monday and Wednesday)
- Transitive closure (last week Friday)
- Partial order relations (**Today**)
- Begin function chapter (Wednesday)

Today:

- Antisymmetry
- Partial order relations
- Topological sort

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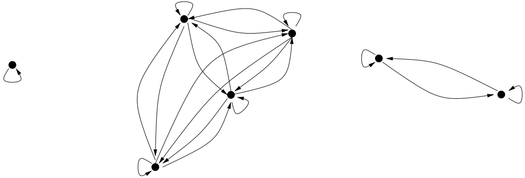

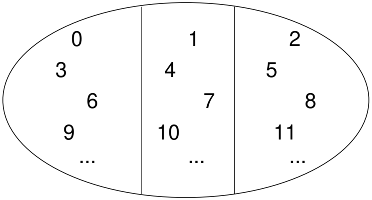
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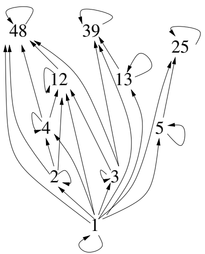
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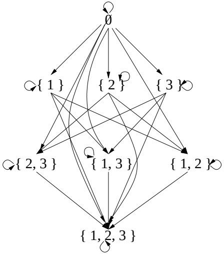
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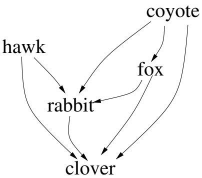
coyote
hawk
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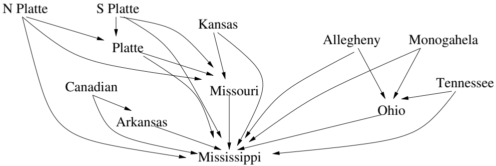
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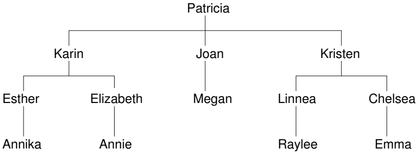
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Annika
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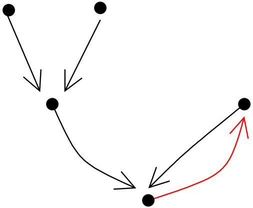
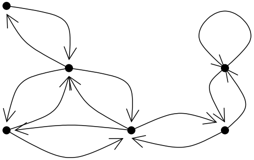
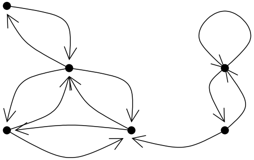


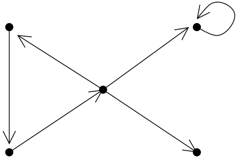


symmetric

All arrows
have a back arrow.



asymmetric
(not symmetric)
There exists an arrow
without a back arrow.



antisymmetric
("very" not symmetric)
No arrows have back arrows
except self loops.

Formal definition:

A relation R on a set X is antisymmetric if $\forall\ x, y \in X$, if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$.

Informal definition:

If both an arrow and its reverse exist in an antisymmetric relation R, then that arrow must be a self loop (and, hence, it is its own reverse).

Alternate formal definition:

A relation R on a set X is antisymmetric if $\forall\ (x, y) \in R$, either $x = y$ or $(y, x) \notin R$.

Rock beats scissors; scissors beats paper; paper beats rock.

Grasshopper eats corn; mouse eats corn; mouse eats grasshopper; snake eats mouse; hawk eats mouse; hawk eats snake.

Aurelia is better than Gwendolyn at pitching; Gwendolyn is better than Aurelia at batting.

Peter Pan is shorter than Treasure Island; Treasure Island is shorter than Anna Karenina; Anna Karenina is shorter than The Count of Monte Christo.

CSCI 235 is a prereq for CSCI 245; CSCI 245 is a rereq for CSCI 345; CSCI 243 is a prereq for CSCI 345; MATH 231 is a prereq for MATH 245; CSCI 345 is a prereq for CSCI 381; MATH 245 is a prereq for CSCI 381.

I married a widow with a grown daughter; my father, a widower, then married my step-daughter. Thus I am my own step-grampa. (The relation in this example is "is biological ancestor of or step-ancestor of".)

A relation R on a set X is antisymmetric if $\forall\ x, y \in X$, if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$.

Ex 5.8.9. Prove that $|$ (divides) on $\mathbb{N}$ is antisymmetric.

Proof. *Suppose $x, y \in \mathbb{N}$, $x|y$, and $y|x$ (that is, $(x, y), (y, x) \in |$). By definition of divides, there exists $i, j \in \mathbb{N}$ such that*

$$\begin{aligned} x &= i \cdot y \\ y &= j \cdot x \end{aligned}$$

Then

$$\begin{aligned} x &= i \cdot j \cdot x && \text{by substitution} \\ 1 &= i \cdot j && \text{by cancellation} \\ i &= j = 1 && \text{by arithmetic} \\ x &= y && \text{by identity} \end{aligned}$$

Therefore $|$ is antisymmetric by definition. $\square$

Antisymmetry:

A relation R on a set X is *antisymmetric* if $\forall\ x, y \in X$, if $(x, y) \in R$ and $(y, x) \in R$, then $x = y$.

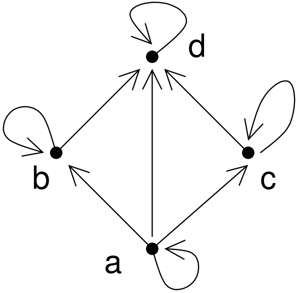
Partial order relation:

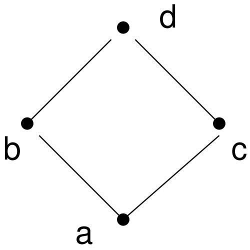
A *partial order relation* (or just *partial order*) is a relation that is reflexive, transitive, and antisymmetric.

A *strict partial order (relation)* is a relation that is irreflexive, transitive and antisymmetric.

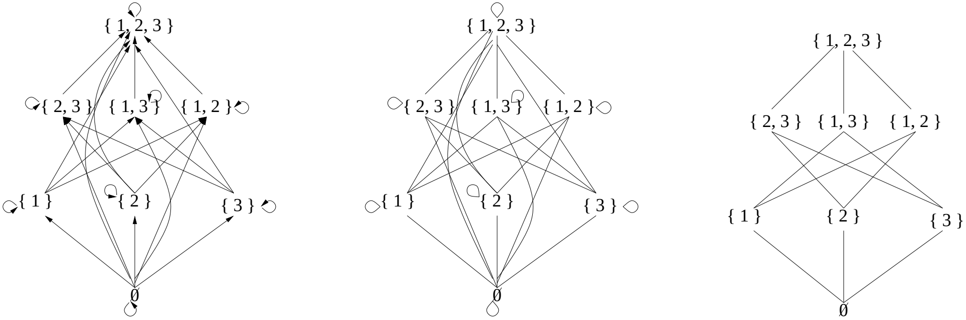
Partially ordered set:

A *partially ordered set* or *poset* is a set together with a partial order on that set.





$$R = \{(a,a),(a,b),(a,c),(a,d),(b,b),(b,d),(c,c),(c,d),(d,d)\}$$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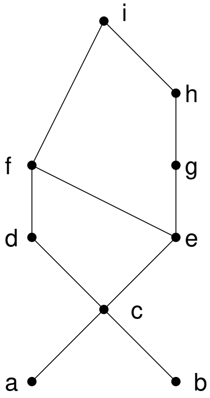


Comparable: $a \preceq c, d \preceq f, e \preceq f, e \preceq h, c \preceq i$

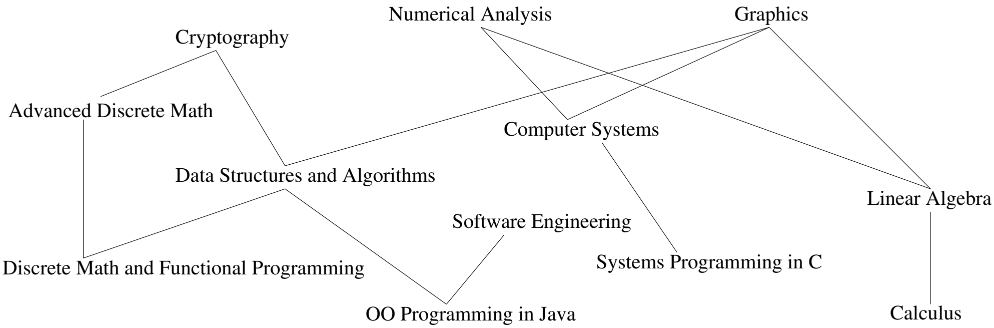
Not comparable: a and b; d and e; f and h

Maximal and greatest: i

Minimal: a and b

No least

Everyday examples: Preparing a meal, writing a term paper, getting dressed


Numerical Analysis
Graphics
Cryptography
Advanced Discrete Math
Computer Systems
Data Structures and Algorithms
Linear Algebra
Software Engineering
Systems Programming in C
Discrete Math and Functional Programming
OO Programming in Java
Calculus

A partial order R on a set X is a *total order* if for all $x, y \in X$, either $x \preceq y$ or $y \preceq x$, that is, x and y are comparable.

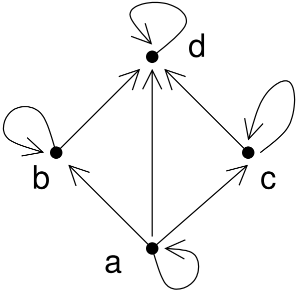
Standard example of a total order: $\leq$.

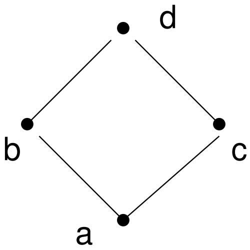
A *partial order relation* (or just *partial order*) is a relation that is reflexive, transitive, and antisymmetric.

A partial order R on a set X is a *total order* if for all $x, y \in X$, either $x \preceq y$ or $y \preceq x$, that is, x and y are comparable.

A *topological sort* of a partial order R is a total order that is a superset of R.

$\mid$ (divides)	$\leq$
is prerequisite for	Ralph takes before
can put on before	you put on before





$R = \{(a,a),(a,b),(a,c),(a,d),(b,b),(b,d),(c,c),(c,d),(d,d)\}$

A topological sort for R: $R \cup \{(b,c)\}$, written as a, b, c, d

Another topological sort for R: $R \cup \{(c,b)\}$, written as a, c, b, d

For next time:

Do Exercises 5.4.(1, 2, 3, 4, 5, 13, 20).

Read Section 6.1