Chapter 3 roadmap:

- Propositions, booleans, logical equivalence. §3.(1 & 2) (Monday)
- Conditional propositions and arguments. §3.(3 & 4) (Wednesday)
- Predicates and quantification. §3.(6 & 7) (**today**)
- Quantified arguments §3.8 (next week Wednesday)
- (Begin proofs next week Friday)

Today:

- Predicates
  - Definition
  - Predicates in set-builder notation
  - Predicates in code
- Quantification
  - Universal quantification
  - Existential quantification
  - Quantification in code
  - Multiple quantification

Four ways to interpret/define the idea of a *predicate*

- A predicate is a proposition with a parameter.

$$x < 5 \qquad\qquad x \text{ is orange}$$

- A predicate is a function whose value is true or false.

$$P(x) = x < 5 \qquad\qquad Q(x) = x \text{ is orange}$$

- A predicate is a part of a sentence that complements a noun phrase to make a proposition.
  A pumpkin is orange.
- A predicate is a truth set

	$P : \mathbb{N} \to \mathbb{B}, P(x) = x < 5$	$Q(x) = x$ is orange
Truth set:	$\{1, 2, 3, 4\}$	{ pumpkin, fall leaves, orange juice, ... }

Universal quantification

"For all multiples of 3, the sum of their digits is a multiple of 3."

Let D be the set of multiples of 3, that is
$D = \{n \in \mathbb{N} \mid n \mathbin{\texttt{mod}} 3 = 0\} = \{3, 6, 9, 12, 15, 18, \ldots\}$

$$\forall\ x \in D, \texttt{sum}(\texttt{digify}(x)) \in D$$

Other examples:

- $\forall x \in \{5, 7, 19, 23, 43\}$, x is prime.
- $\forall x \in \{4, 16, 25, 31\}$, x is a perfect square.

Existential quantification

"There is a multiple of 3 that is not a perfect square."

$$\exists\ x \in D \mid x \text{ is not a perfect square}$$

Alternately, "Some multiples of 3 are not perfect squares."

General forms for universal and existential quantification:

$$\forall\ x \in X, P(x) \qquad\qquad \exists\ x \in X \mid P(x)$$

$\forall\ x \in \emptyset, P(x)$ is **always (vacuously) true**.

$\exists\ x \in \emptyset \mid P(x)$ is **always false**

$$
\begin{aligned}
&\sim (\forall\ x \in X, P(x)) \\
&\quad \equiv \quad \sim (P(x_1) \wedge P(x_2) \wedge \cdots) \\
&\quad \equiv \quad \sim P(x_1) \vee \sim P(x_2) \vee \cdots \quad \text{By DeMorgan's Law} \\
&\quad \equiv \quad \exists\ x \in X \mid\ \sim P(x)
\end{aligned}
$$

For next time:

Do Exercises 3.6.(2, 5-9) and 3.7.(20-24).
(All programming)

Read Section 3.8

Take quiz