Chapter 3:

- Propositions, booleans, logical equivalence. §3.(1 & 2) (**Today**)
- Conditional propositions and arguments. §3.(3 & 4) (Wednesday)
- Predicates and quantification. §3.(6 & 7) (Friday)
- Quantified arguments §3.8 (next week Wednesday)
- (Begin proofs next week Friday)

Today:

- Highlight main points of §3.1: Propositions, forms, etc
- Take a programming break
- Work through §3.2: Logical equivalences (Game 1)

A **proposition** is a sentence that is true or false, but not both.

It is snowing and it is not Thursday.

A **propositional form** is like a proposition but with content replaced by variables.

p and not q

$p \wedge \sim q$

$$\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$$

$$+ - \times \div$$

$\times$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	4	6
3	0	3	6	9

$$\mathbb{B} = \{T, F\}$$

$$\vee \wedge \sim$$

$\wedge$	T	F
T	T	F
F	F	F

$\wedge$	T	F
T	T	F
F	F	F

$\vee$	T	F
T	T	T
F	T	F

p	$\sim p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \wedge q$	$p \vee q$	$\sim p$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Evaluate (to T or F) this logical expression:

$$(T \wedge (\sim F \vee F)) \wedge (T \wedge T)$$

Evaluate (to T or F) this logical expression:

$(T \vee F) \wedge \sim (F \wedge T)$

Evaluate (to T or F) this logical expression:

$(F \vee F \vee T) \wedge (\sim T \wedge F)$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim (p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Commutative laws:	$p \wedge q$	$\equiv$	$q \wedge p$	$p \vee q$	$\equiv$	$q \vee p$
Associative laws:	$(p \wedge q) \wedge r$	$\equiv$	$p \wedge (q \wedge r)$	$(p \vee q) \vee r$	$\equiv$	$p \vee (q \vee r)$
Distributive laws:	$p \wedge (q \vee r)$	$\equiv$	$(p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r)$	$\equiv$	$(p \vee q) \wedge (p \vee r)$
Absorption laws:	$p \wedge (p \vee q)$	$\equiv$	p	$p \vee (p \wedge q)$	$\equiv$	p
Idempotent laws:	$p \wedge p$	$\equiv$	p	$p \vee p$	$\equiv$	p
Double negative law:	$\sim\sim p$	$\equiv$	p			
DeMorgan's laws:	$\sim (p \wedge q)$	$\equiv$	$\sim p \vee \sim q$	$\sim (p \vee q)$	$\equiv$	$\sim p \wedge \sim q$
Negation laws:	$p \vee \sim p$	$\equiv$	T	$p \wedge \sim p$	$\equiv$	F
Universal bound laws:	$p \vee T$	$\equiv$	T	$p \wedge F$	$\equiv$	F
Identity laws:	$p \wedge T$	$\equiv$	p	$p \vee F$	$\equiv$	p
Tautology and contradiction laws:	$\sim T$	$\equiv$	F	$\sim F$	$\equiv$	T

Remember from high school algebra that there are "simplify" problems and "solve" problems.

- Simplify $3x(2+3x)^2+1$.

$$\begin{aligned} & 3x(2+3x)^2+1 \\ = \quad & 3x(4+12x+9x^2)+1 \\ = \quad & 12x+36x^2+27x^3+1 \\ = \quad & 27x^3+36x^2+12x+1 \end{aligned}$$

- Solve $12x = 57 - 7x$ for x.

$$\begin{aligned} 12x &= 57-7x \\ 19x &= 57 \\ x &= 3 \end{aligned}$$

Suppose we were to show that $\sim(\sim p \wedge q) \vee (p \vee \sim p) \equiv p \vee \sim q$.

Do this:

$$
\begin{aligned}
&\sim(\sim p \wedge q) \vee (p \wedge \sim p) && \\
\equiv\ & \sim(\sim p \wedge q) \vee F && \text{by negation law} \\
\equiv\ & \sim(\sim p \wedge q) && \text{by identity law} \\
\equiv\ & p \vee \sim q && \text{by De Morgan's}
\end{aligned}
$$

Don't do this:

$$
\begin{aligned}
\sim(\sim p \wedge q) \vee (p \wedge \sim p) &\equiv p \vee \sim q && \\
\sim(\sim p \wedge q) \vee F &\equiv p \vee \sim q && \text{by negation law} \\
\sim(\sim p \wedge q) &\equiv p \vee \sim q && \text{by identity law} \\
p \vee \sim q &\equiv p \vee \sim q && \text{by De Morgan's}
\end{aligned}
$$

For next time:

Do Exercises 3.2.(2, 4, 8-12)

Read 3.(1 & 2) if you haven't already
(3.3 hasn't been written yet. . .)
Read 3.(4 & 5)
Take quiz