Semester roadmap:

Ch 1: Sets
Ch 2: Sequences
—Test 1, Feb 7 —
Ch 3: Symbolic logic
Ch 4: Proofs
— Test 2, Mar 5 —
Ch 5: Relations
Ch 6: Functions
— Test 3, Apr 11 —
Ch 7: Self reference

Chapter 7 roadmap:

- Recursively-defined sets (**Today**)
- Recursive proofs I: Structural induction (Wednesday)
- Recursive proofs II: Mathematical induction (next week Monday)
- Non-recursive programs—loops (next week Wednesday)
- Recursive proofs III: Loop invariants (next week Friday)
- Recursively-defined sets application: The Huffman encoding (week-after Monday)
- Leftover topic: Arrays, vectors, and intervals (week-after Wednesday)

### Axiom 7
*There exists a whole number 0.*

### Axiom 8
*Every whole number n has a successor,* $\texttt{succ}\ n$.

### Axiom 9
*No whole number has 0 as its successor.*

### Axiom 10
*If* $a, b \in \mathbb{W}$, *then* $a = b$ *iff* $\texttt{succ}\ a = \texttt{succ}\ b$.

A *whole number* is either zero or one more than another whole number.

Compare to:
A *pizza* is either crust or a topping together with its sub-pizza.

5 is a whole number because

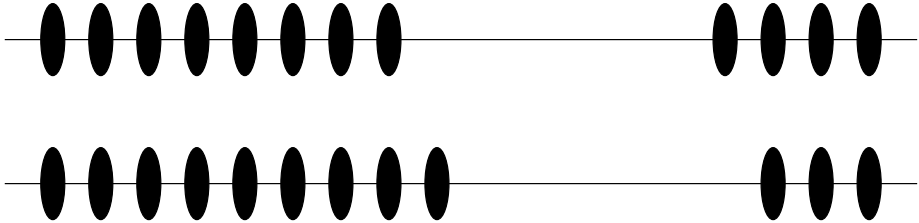
5 is a whole number because it is the successor of 4, which is a whole number because

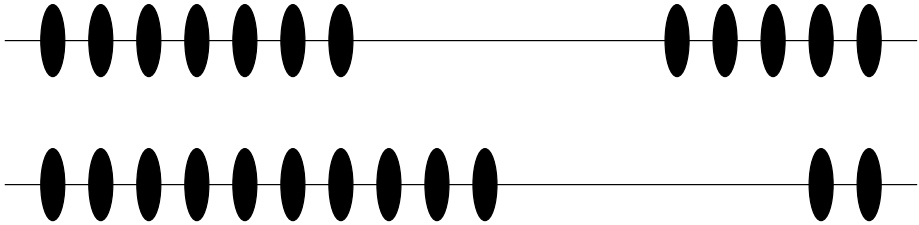
5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because

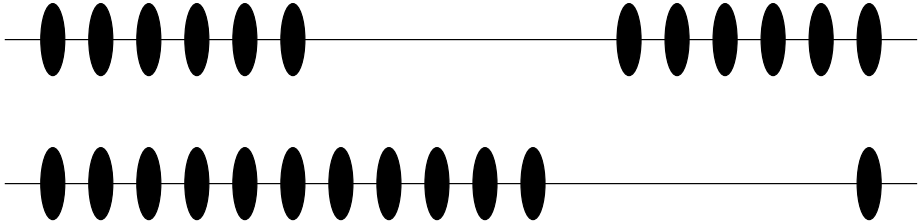
5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because it is the successor of
2, which is a whole number because

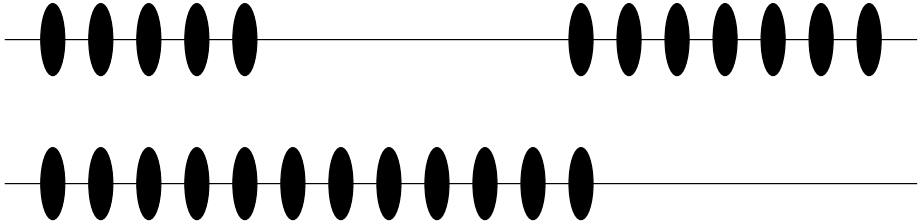
5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because it is the successor of
2, which is a whole number because it is the successor of
1, which is a whole number because

5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because it is the successor of
2, which is a whole number because it is the successor of
1, which is a whole number because it is the successor of
0, which is a whole number by Axiom 7.





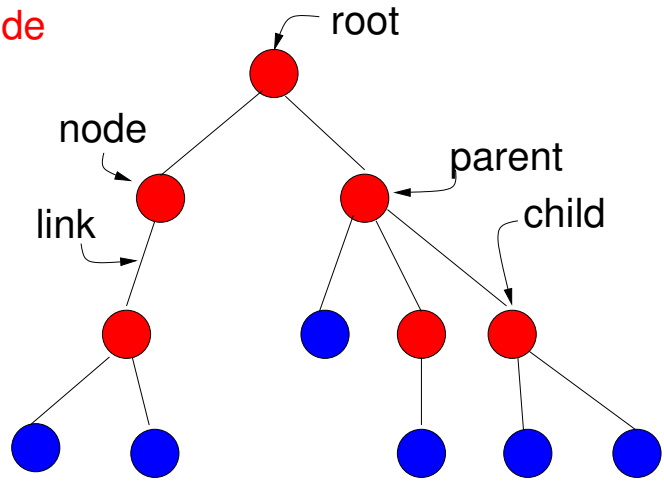




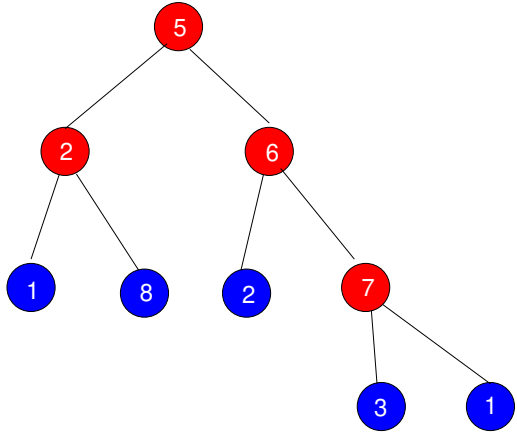
# Tree

internal node

leaf



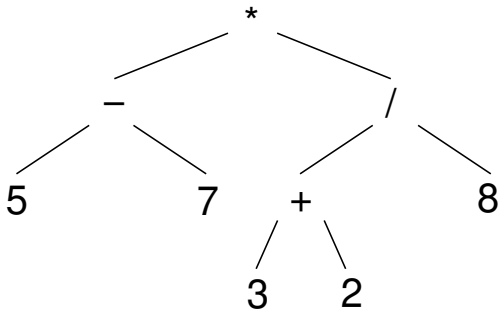
# Full Binary Tree



Expression trees:

$$\begin{aligned} \textit{Expression} &\rightarrow \textit{Variable} \mid \textit{Constant} \\ &\quad \mid (\ \textit{Expression}\ \textit{Operator}\ \textit{Expression}\ ) \end{aligned}$$

$$\textit{Operator} \rightarrow + \mid - \mid * \mid /$$



**For next time:**

*Read Section 7.1 (to review what we did today)*
*and Section 7.2 (for what we're doing next time)*

*Do Exercises 7.1.(1-5)*

*(No quiz)*