Chapter 7 outline:

- Recursively-defined sets (week-before Monday)
- Structural induction (week-before Wednesday)
- Mathematical induction (last week Monday)
- Non-recursive programs—loops (last week Wednesday)
- Mathematical-induction application: Loop invariant proofs (last week Friday)
- Recursively-defined sets application: The Huffman encoding (Monday)
- Leftover topic: Arrays, vectors, and intervals (**Today**)

Today:

- Sequences and sequence-like mathematical objects
  - Vectors
  - Intervals
  - Matrices
- Python arrays (ndarray)
  - As vectors
  - As intervals
  - As matrices
  - As data sets

A **sequence** *is an ordered collection of elements.*

A **sequence** *over set X with index set $\mathcal{I}$ is a function from $\mathcal{I}$ to X. Thus the function labels the elements in the set X using elements from $\mathcal{I}$.*

The index set must have a total order and must be countable.

The sequence

$$[1, 2, 4, 8, 16 \ldots]$$

is defined as the function $f : \mathbb{W} \to \mathbb{N}, f(x) = 2^x$.

$$\begin{array}{rrrcr} 3x_0 & +2x_1 & -\frac{1}{2}x_2 & = & -\frac{4}{3} \\ -4x_0 & -x_1 & +10x_2 & = & 5 \\ \frac{1}{8}x_0 & +\frac{2}{3}x_1 & +x_2 & = & 7 \end{array}$$

$$A = \begin{pmatrix} 3 & 2 & -\frac{1}{2} \\ -4 & -1 & 10 \\ \frac{1}{8} & \frac{2}{3} & 1 \end{pmatrix}$$

$$A\vec{x} = \vec{b} \quad \text{or} \quad \begin{pmatrix} 3 & 2 & -\frac{1}{2} \\ -4 & -1 & 10 \\ \frac{1}{8} & \frac{2}{3} & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\frac{4}{3} \\ 5 \\ 7 \end{pmatrix}$$