

Chapter 1 outline:

- ▶ Introduction, sets and elements (last week Monday)
- ▶ Python expressions (last week Wednesday)
- ▶ Python functions; denoting sets (last week Friday)
- ▶ Set operations; visual verification of set propositions (Wednesday)
- ▶ Cardinality, Cartesian products, powersets (**today**)
- ▶ (Begin Chapter 2 Sequence next week)

Today:

- ▶ Cardinality
- ▶ Disjoint[edness], pairwise disjoint[edness], partitions
- ▶ Cartesian products
- ▶ Powersets

term	grammar	informal definition	formal definition
Cardinality	noun	The cardinality of a set is the number of elements in that set.	See Chapter 6.
Disjoint	adjective	Two sets are disjoint if they have no elements in common.	X and Y are disjoint if $X \cap Y = \emptyset$.
Pairwise disjoint	adjective	A collection of sets are pairwise disjoint if no two of them have any elements in common.	The sets X_1, X_2, \dots, X_n are pairwise disjoint if for any two sets X_i and X_j , where $i \neq j$, $X_i \cap X_j = \emptyset$.
Partition	noun	A collection of subsets of a set are a partition of that set if they are pairwise disjoint and together make up the entire set.	If X is a set, then a partition of X is a set of sets $\{X_1, X_2, \dots, X_n\}$ such that X_1, X_2, \dots, X_n are pairwise disjoint and $X_1 \cup X_2 \cup \dots \cup X_n = X$.

Compute the cardinality:

$$|\{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6\}|$$

$$|[0, \pi) \cap \mathbb{Z}|$$

$$|FacultyInThisRoom - StudentsInThisRoom|$$

Which are disjoint?

\mathbb{Z} and \mathbb{R}

\mathbb{Z} and \mathbb{R}^-

$[0, 5)$ and $[5, 10)$

Plants and *Fungi*

MathClasses and *CSCIClasses*

DeciduousTrees and *ConiferousTrees*

1.7.1 What is the cardinality of $\{0, 1, 2, \dots, n\}$?

1.7.5 One might be tempted to think $|A \cup B| = |A| + |B|$, but this is not true in general. Why not? (Assume A and B are finite.)

1.7.6 Is $|A - B| = |A| - |B|$ true in general? If so, explain why. If not, under what special circumstances is it true? (Assume A and B are finite.)

1.7.7 Consider the sets $\{1, 2, 3\}$, $\{2, 3, 4\}$, $\{3, 4, 5\}$, and $\{4, 5, 6\}$. Notice that

$$\{1, 2, 3\} \cap \{2, 3, 4\} \cap \{3, 4, 5\} \cap \{4, 5, 6\} = \emptyset$$

Are these sets pairwise disjoint?

1.7.8 Describe three distinct partitions of the set \mathbb{Z} , apart from the partitions given in this section.

Describe the following Cartesian product: $\{-1, 0, 1\} \times \{a, b\}$

1.7.12 If A and B are finite sets, what is $|A \times B|$ in terms of $|A|$ and $|B|$?

1.7.13 Based on our description of the real number plane as a Cartesian product, explain how a line can be interpreted as a set.

1.7.14 Explain how \mathbb{C} , the set of complex numbers, can be thought of as a Cartesian product.

1.7.15 Any rational number (an element of set \mathbb{Q}) has two integers as components. Why not rewrite fractions as ordered pairs (for example, $\frac{1}{2}$ as $(1, 2)$ and $\frac{3}{4}$ as $(3, 4)$) and claim that \mathbb{Q} can be thought of as $\mathbb{Z} \times \mathbb{Z}$? Explain why these two sets *cannot* be thought of as two different ways to write the same set. (There are at least two reasons.)

Which are true?

$$\{3\} \in \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$3 \in \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$\{3\} \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$3 \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\})$$

$$a \in A \text{ iff } \{a\} \in \mathcal{P}(A)$$

$$a \in A \text{ iff } \{a\} \subseteq \mathcal{P}(A)$$

$$A \subseteq B \text{ iff } A \subseteq \mathcal{P}(B)$$

$$A \subseteq B \text{ iff } A \in \mathcal{P}(B)$$

Which are true?

$$\{A\} \subseteq \mathcal{P}(A)$$

$$A \in \mathcal{P}(A)$$

$$\{A\} \in \mathcal{P}(A)$$

$$\mathbb{Z} \in \mathcal{P}(\mathbb{R})$$

$$\emptyset \in \mathcal{P}(A)$$

$$\emptyset = \mathcal{P}(\emptyset)$$

Note that

- ▶ $a \in A$ iff $\{a\} \in \mathcal{P}(A)$
- ▶ $A \subseteq B$ iff $A \in \mathcal{P}(B)$
- ▶ $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$
- ▶ $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$

Observe

$$\begin{aligned}\mathcal{P}(\{1, 2, 3\}) &= \{ \emptyset \\ &\quad \{1\}, \{2\}, \{3\} \\ &\quad \{1, 2\}, \{1, 3\}, \{2, 3\} \\ &\quad \{1, 2, 3\} \} \\ &= \{ \{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\} \\ &\quad \emptyset, \{2\}, \{3\}, \{2, 3\} \} \\ &= \mathcal{P}(\{2, 3\}) \cup \left[\begin{array}{c} \text{1 added to each set} \\ \text{of } \mathcal{P}(\{2, 3\}) \end{array} \right] = \mathcal{P}(\{2, 3\}) \cup \\ &\quad \{ \{1\} \cup X \mid X \in \mathcal{P}(\{2, 3\}) \} \end{aligned}$$

If $a \in A$, then $\mathcal{P}(A) = \mathcal{P}(A - \{a\}) \cup \{ \{a\} \cup X \mid X \in \mathcal{P}(A - \{a\}) \}$

What is $|\mathcal{P}(X)|$ in terms of $|X|$?

For next time:

Pg 48–50: 1.7.(2, 3, 4, 11, 20, 21, 23)

Pg 43: 1.8.(2, 11, 14)

Read 2.1

Take quiz