Chapter 5, Binary search trees:

- Binary search trees; the balanced BST problem (spring-break eve; finished last week Monday)
- AVL trees (last week Monday and Wednesday)
- Traditional red-black trees (last week Friday, finish **Today**)
- Left-leaning red-black trees (**Today**)
- "Wrap-up" BST (Wednesday)
- Begin dynamic programming (Friday)

Today:

- Finishing Traditional RB
- LLRB context and definition
- LLRB invariant and cases
- Performance comparison among AVL, TrRB, and LLRB

Why invariants?

- An invariant is a constraint we put on our code to help us guarantee something about it.
- The general invariant for BSTs guarantees the correctness of our find algorithm.
- The invariants for AVL trees and RB trees guarantee logarithmic-time operations.

A stronger constraint is both a stronger constraint to *maintain* and a stronger constraint to *assume*.

A left-leaning red-black tree is a binary tree (usually a BST) that is either empty or it is rooted at node T such that

- T is either red or black.
- Both of T's children are roots of left-leaning red-black trees.
- T's right child is black.
- If T is red, then its left child is black.
- The left-leaning red-black trees rooted at its children have equal blackheight; moreover, the blackheight of the tree rooted at T is one more that the blackheight of its children if T is black or equal to that of its chidren if T is red.

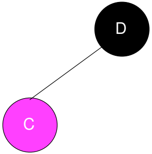
Left-leaning

D

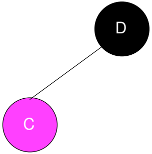
Traditional

D

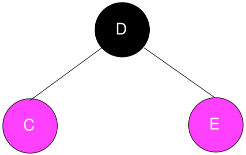
Left-leaning



Traditional



Left-leaning



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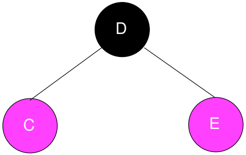
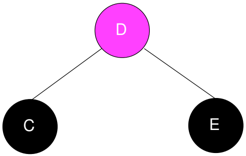


Left-leaning



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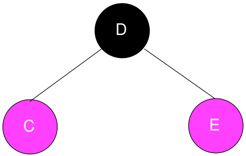

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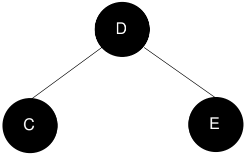

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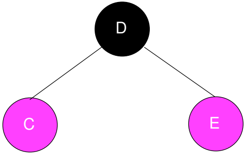

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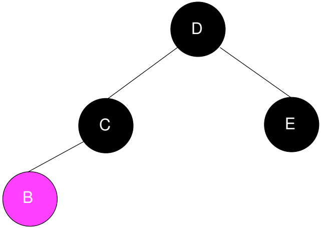

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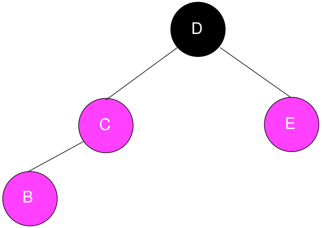

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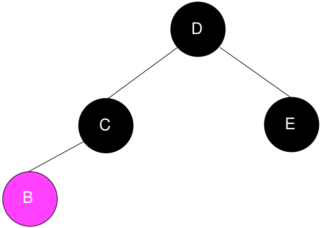

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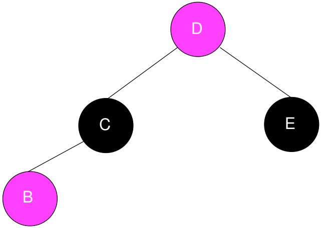

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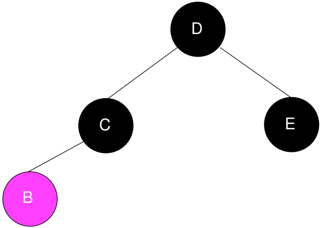

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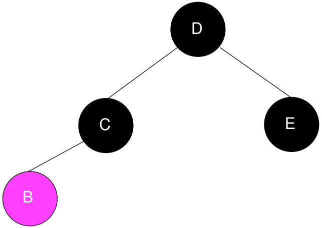

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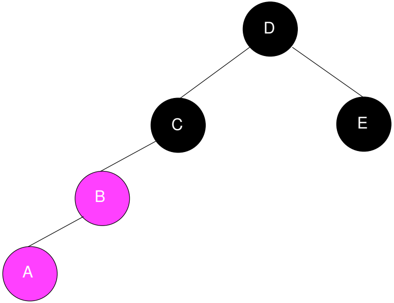

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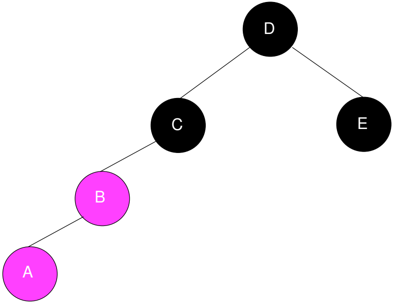

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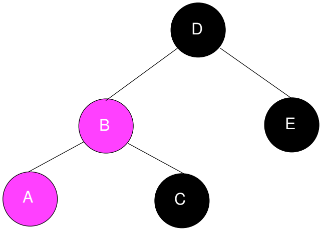

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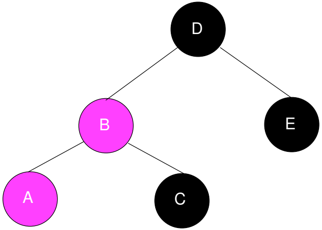
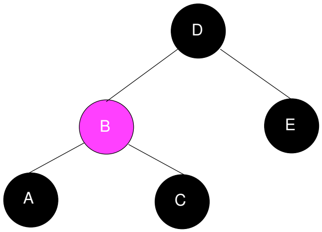


Left-leaning



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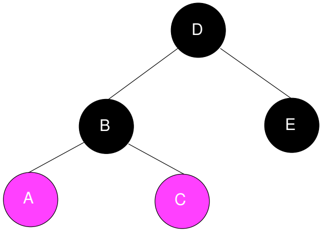


Potential violations

$$\left.\begin{array}{l}\text{Ignorant node}\\[1em]\text{Inconsistent backheight}\\[1em]\text{Red null}\end{array}\right\}\text{shouldn't happen}$$

$$\left.\begin{array}{l}\text{Double red}\\[1em]\text{Right red}\end{array}\right\}\text{fix when they happen}$$

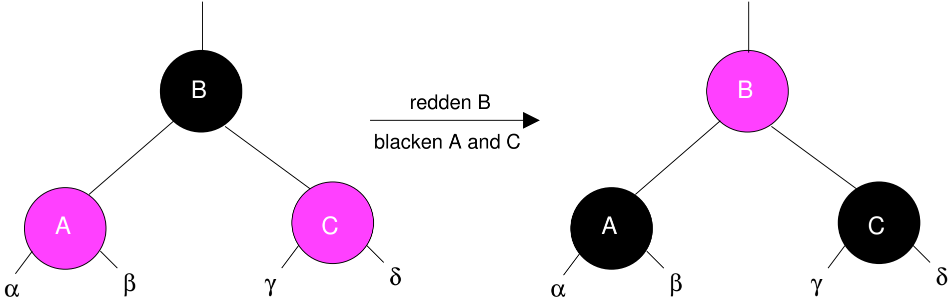
Invariant 28 (Postconditions of `RealNode.put()` with `LLRBBalancer`.) Let x be the root of a subtree on which `put()` is called and let y be the node returned, that is, the root of the resulting subtree.

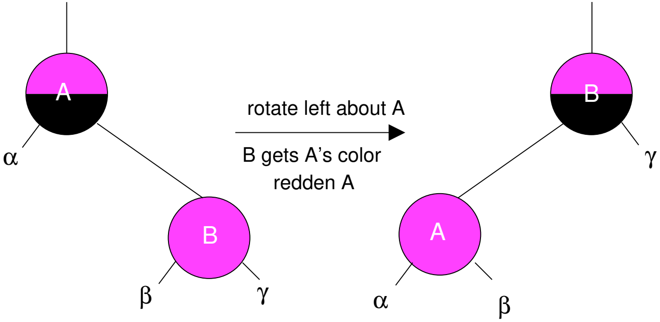
(a) The subtree rooted at y has a consistent black height.

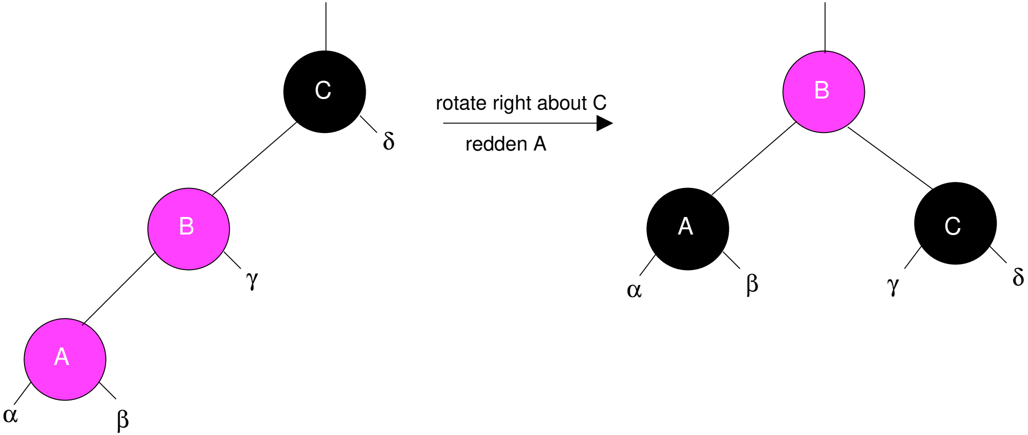
(b) The black height of subtree rooted at y is equal to the original black height of the subtree rooted at x.

(c) The subtree rooted at y has no double-red violations except, possibly, both y and its left child is red, which can happen only if x is a left child.

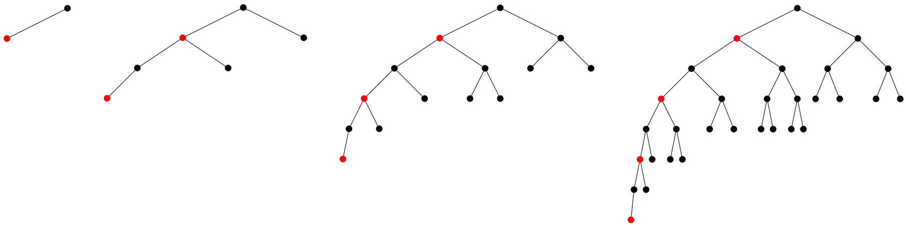
(d) The subtree rooted at y has no right-red violations.


B
A
C
α
β
γ
δ
redden B
blacken A and C
B
A
C
α
β
γ
δ

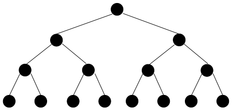

A
α
B
β
γ
rotate left about A
B gets A's color
redden A
B
γ
A
α
β


C
δ
B
γ
A
α
β
rotate right about C
redden A
B
A
C
α
β
γ
δ

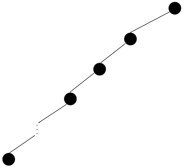
Blackheight	1	2	3	4



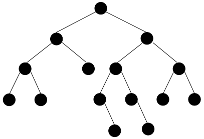
Height	2	4	6	8
Nodes	2	6	14	30



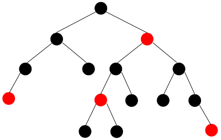
Height: 3
Leaves: 8
Total depth: 34



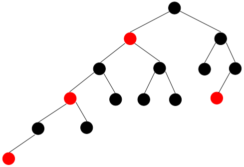
Height: 14
Leaves: 1
Total depth: 105



Height: 4
Leaves: 7
Total depth: 36



Height: 4
Leaves: 7
Total depth: 37



Height: 5
Leaves: 7
Total depth: 38

	After puts			After removals		
	Height	Leaf %	Total depth	Height	Leaf %	Total depth
Unbalanced	32	33.3%	134507	28	16.8%	61207
	31	33.2%	127865	26	17.0%	58171
	30	33.1%	129037	26	16.9%	58610
	28	33.5%	124463	26	17.3%	56086
	32	33.4%	136730	28	16.9%	62092
AVL	16	43.2%	100327	14	21.5%	46088
	15	42.9%	100395	14	21.1%	46028
	15	42.8%	100341	14	21.1%	46028
	15	42.8%	100282	14	21.3%	45973
	15	43.0%	100582	14	21.2%	46097
Traditional RB	16	42.8%	101948	16	21.5%	46729
	16	42.9%	101226	15	21.4%	46344
	16	43.1%	101525	15	21.5%	46462
	16	42.7%	101680	16	21.5%	46572
	16	42.9%	101292	15	21.4%	46338
Left-leaning RB	18	42.8%	102288	18	21.6%	46950
	19	42.9%	102860	16	21.3%	46774
	18	43.1%	101949	17	21.5%	46691
	18	42.7%	102011	17	21.6%	46938
	19	42.9%	102552	16	21.4%	46764