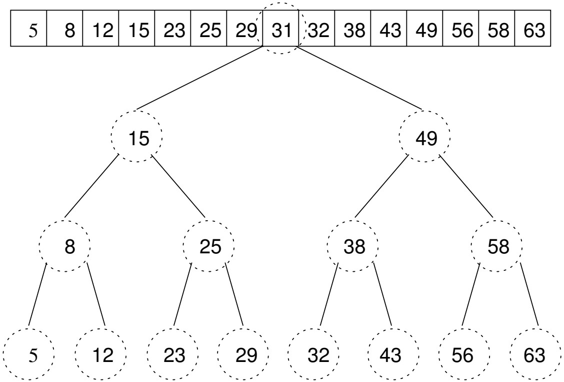
Chapter 5, Binary search trees:

- Binary search trees; the balanced BST problem (spring-break eve; finished last week Monday)
- AVL trees (last week Monday and Wednesday)
- Traditional red-black trees (last week Friday, finished Monday)
- Left-leaning red-black trees (Monday, finish **Today**)
- "Wrap-up" BST (**Today**)
- Begin dynamic programming (Friday)
- Test 2 Thursday, Apr 10

Today:

- Balanced tree comparisons
- Survey of B-trees


5 8 12 15 23 25 29 31 32 38 43 49 56 58 63
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49
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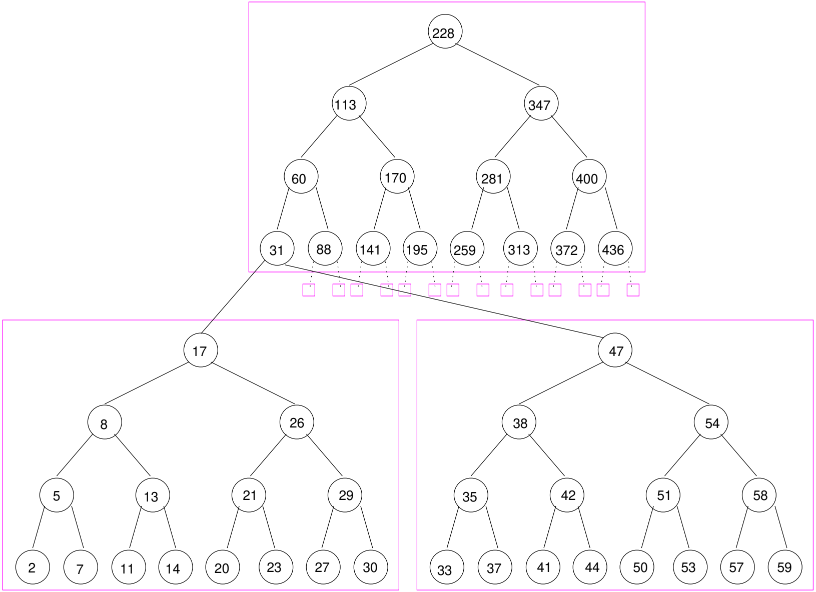
228
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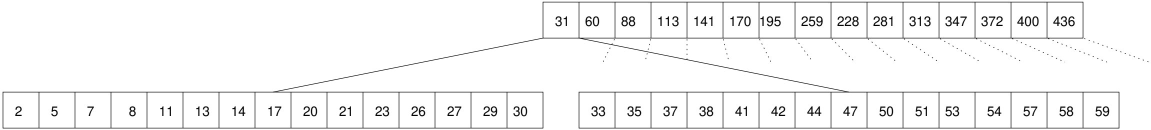
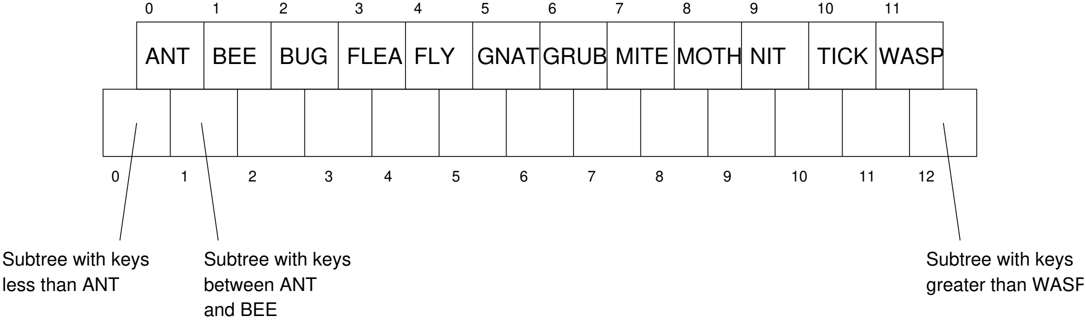
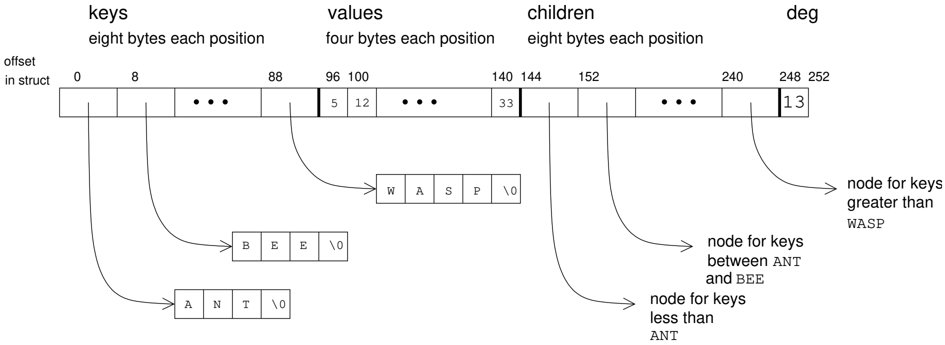

31 60 88 113 141 170 195 259 228 281 313 347 372 400 436
2 5 7 8 11 13 14 17 20 21 23 26 27 29 30
33 35 37 38 41 42 44 47 50 51 53 54 57 58 59

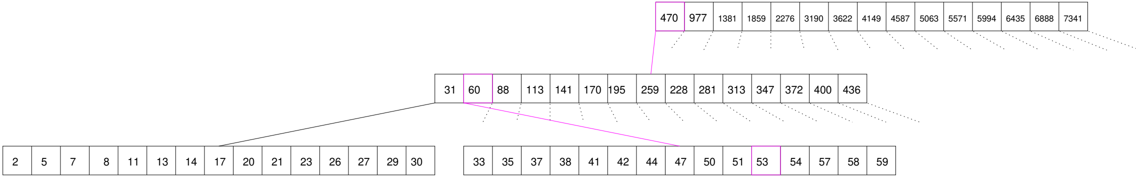


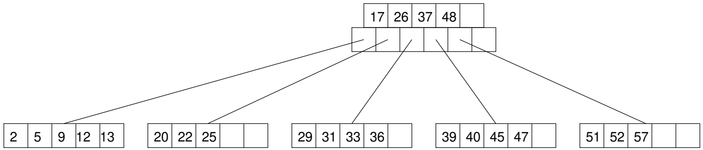
0 1 2 3 4 5 6 7 8 9 10 11
ANT BEE BUG FLEA FLY GNAT GRUB MITE MOTH NIT TICK WASP
0 1 2 3 4 5 6 7 8 9 10 11 12
Subtree with keys less than ANT
Subtree with keys between ANT and BEE
Subtree with keys greater than WASP

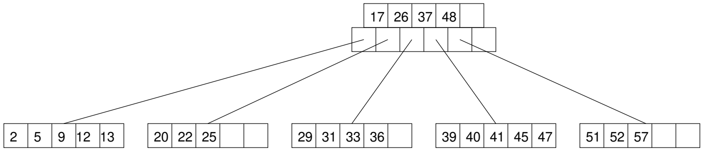
Formally, a B-tree with maximum degree M over some ordered key type is either

- empty, or
- a node with with $d - 1$ keys and d children, designated as lists `keys` and `children` such that
  - $\lceil M/2 \rceil \leq d \leq M$,
  - `children`[0] is a B-tree such that all of the keys in that tree are less than `keys`[0],
  - for all $i \in [1, d - 1)$, `children`$[i]$ is a B-tree such that all of the keys in that tree are greater than `keys`$[i - 1]$ and less than `keys`$[i]$,
  - and `children`$[d - 1]$ is a B-tree such that all of the keys in that tree are greater than `keys`$[d - 2]$.

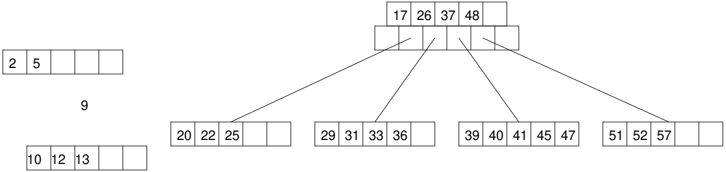

keys
eight bytes each position
values
four bytes each position
children
eight bytes each position
deg
offset in struct
0
8
88
96
100
140
144
152
240
248
252
5
12
33
13
W A S P \0
B E E \0
A N T \0
node for keys greater than WASP
node for keys between ANT and BEE
node for keys less than ANT


470 977 1381 1859 2276 3190 3622 4149 4587 5063 5571 5994 6435 6888 7341
31 60 88 113 141 170 195 259 228 281 313 347 372 400 436
2 5 7 8 11 13 14 17 20 21 23 26 27 29 30
33 35 37 38 41 42 44 47 50 51 53 54 57 58 59


17 26 37 48
2 5 9 12 13
20 22 25
29 31 33 36
39 40 45 47
51 52 57



17 26 37 48
2 5 9 12 13
20 22 25
29 31 33 36
39 40 41 45 47
51 52 57



17 26 37 48
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29 31 33 36
39 40 41 45 47
51 52 57
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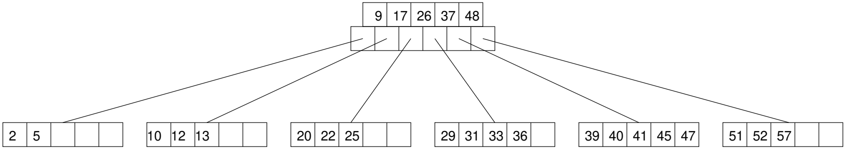
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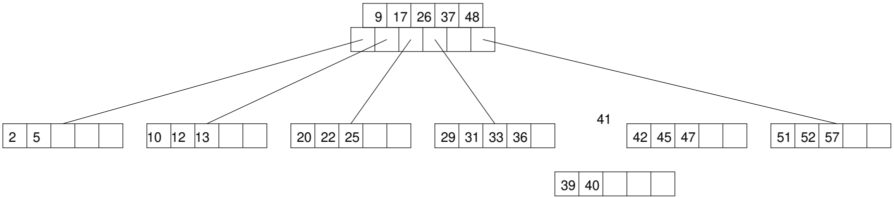
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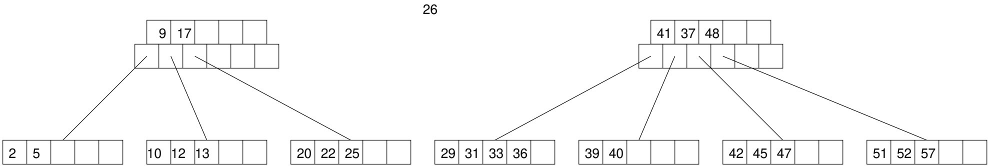
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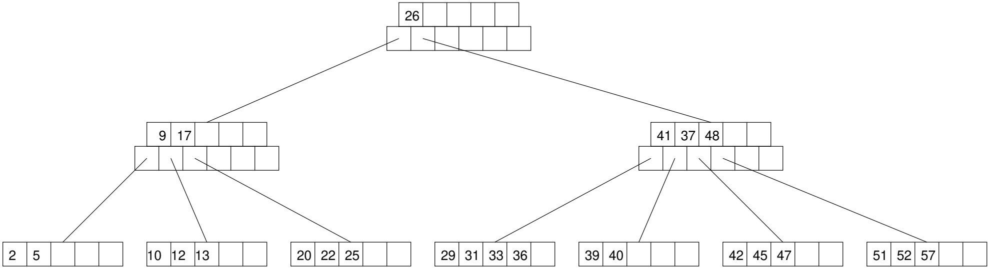
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20 22 25
29 31 33 36
39 40
42 45 47
51 52 57

$$\underbrace{(M-1)}_{\text{keys per node}} \underbrace{\sum_{i=0}^{h-1} M^i}_{\text{sum of nodes at each level}} = (M-1)\frac{M^h - 1}{M-1} = M^h - 1$$

$$
\begin{aligned}
n &= M^h - 1 \\
M^h &= n + 1 \\
h &= \log_M(n+1)
\end{aligned}
$$

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$$

$$h = \log_{\frac{M}{2}}(n+1) = \frac{\log_M(n+1)}{1 - \log_M 2}$$

Cost of a search:

$$
\begin{aligned}
\lg M \cdot h &= \lg M \cdot \frac{\log_M(n+1)}{1-\log_M 2} \\
&= \lg M \frac{\frac{\lg(n+1)}{\lg M}}{1-\frac{\lg 2}{\lg M}} \\
&= \frac{\lg(n+1)}{1-\frac{1}{\lg M}} \\
&= \frac{\lg M}{\lg M - 1} \lg(n+1)
\end{aligned}
$$

Compare: $1.44 \lg n$ for AVL trees, $2 \lg n$ for RB trees.

Let c_0 be the cost of searching at a node (proportional to $\lg M$) and c_1 be the cost of reading a node from memory. The the cost of an entire search is

$$(c_0 + c_1)\frac{\log_M(n+1)}{1 - \log_M 2}$$

Now, consolidate the constants by letting $d = \frac{c_0+c_1}{1-\log_M 2}$, and we have

$$d \log_M(n+1)$$

Coming up:

Do **BST rotations** *project (due Wed, Mar 19)—nothing to turn in*
Do **AVL trees** *project (due Mon, Mar 24)*
Do **Traditional RB** *project (due Fri, Mar 28)*

Due **Wed, Mar 26** *(end of day)—but spread it out*
Read Sections 5.(4-6)
Do Exercise 5.13
Take quiz (red-black trees)

Due **Mon, Mar 31** *(end of day)*
Read Section 6.(1&2)
Do Exercises 6.(5–7)
Take quiz

Due **Tues, Apr 1** *(end of day)*
Read Section 6.3
Do Exercises 6.(16, 19, 23, 33)
Take quiz