Chapter 3 roadmap:

- Propositions, boolean logic, logical equivalences. **Game 1** (Monday)
- Conditional propositions. **SML** (Wednesday)
- Arguments. **Game 2** (Today)
- Predicates and quantification. **SML** (next week Monday)
- Quantified arguments. **Game 3** (next week Wednesday)
- Review for test (next week Friday)

Today:

- Define arguments
- Consider known “syllogisms”
- Practice verifying argument forms (Game 2)
Valid argument

If it is Monday, then it is raining.
It is Monday.
Therefore it is raining.

\[
\begin{array}{c|c}
 p & q \\
\hline
 T & T \\
 T & F \\
 F & T \\
 F & F \\
\end{array}
\]

\[
\begin{array}{c|c|c}
 p & p \rightarrow q & q \\
\hline
 T & T & T \\
 T & F & F \\
 F & T & T \\
 F & F & F \\end{array}
\]

Critical row
Invalid argument

If it is raining, then there are clouds.
There are clouds.
Therefore it is raining.

\[ p \rightarrow q \]
\[ q \]
\[ \therefore p \]
### Alternate definition of validity

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \rightarrow q$</th>
<th>$(p \land (p \rightarrow q)) \rightarrow q$</th>
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</thead>
<tbody>
<tr>
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<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \rightarrow q$</th>
<th>$(q \land (p \rightarrow q)) \rightarrow p$</th>
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</table>
Modus tollens

If it is spring, then the daffodils bloom.
The daffodils aren’t blooming.
Therefore it is not spring.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \rightarrow q$</th>
<th>$\sim q$</th>
<th>$\sim p$</th>
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<td>$T$</td>
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<tr>
<td>Rule</td>
<td>Premise 1</td>
<td>Premise 2</td>
<td>Conclusion</td>
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<tr>
<td><strong>Modus Ponens</strong></td>
<td>$p \rightarrow q$</td>
<td>$p$</td>
<td>$\therefore q$</td>
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<tr>
<td><strong>Modus Tollens</strong></td>
<td>$p \rightarrow q$</td>
<td>$\sim q$</td>
<td>$\therefore \sim p$</td>
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<tr>
<td><strong>Generalization</strong></td>
<td>$p$</td>
<td></td>
<td>$\therefore p \lor q$</td>
<td></td>
</tr>
<tr>
<td><strong>Specialization</strong></td>
<td>$p \land q$</td>
<td></td>
<td>$\therefore p$</td>
<td></td>
</tr>
<tr>
<td><strong>Elimination</strong></td>
<td>$p \lor q$</td>
<td>$\sim p$</td>
<td>$\therefore q$</td>
<td></td>
</tr>
<tr>
<td><strong>Transitivity</strong></td>
<td>$p \rightarrow q$</td>
<td>$q \rightarrow r$</td>
<td>$\therefore p \rightarrow r$</td>
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<td></td>
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<td></td>
<td>$\therefore p \rightarrow r$</td>
<td></td>
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<tr>
<td><strong>Division into cases</strong></td>
<td>$p \lor q$</td>
<td>$p \rightarrow r$</td>
<td>$\therefore q \rightarrow r$</td>
<td></td>
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<td></td>
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<td></td>
<td>$\therefore r$</td>
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<tr>
<td><strong>Contradiction</strong></td>
<td>$p \rightarrow F$</td>
<td></td>
<td>$\therefore \sim p$</td>
<td></td>
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</tbody>
</table>
Syllogisms in literature

Elmination:

If anyone knows anything about anything, it’s Owl who knows something about something, or my name isn’t Winnie-the-Pooh. Which it is. So there you have it.

A. A. Milne, Winnie-the-Pooh, Ch 4.

Division into cases:

Soon her eye fell on a little glass box that was lying under the table: she opened it, and found in it a very small cake, on which the words “EAT ME” were beautifully marked in currants. “Well, I’ll eat it,” said Alice, “and if it makes me grow larger, I can reach the key; and if it makes me grow smaller, I can creep under the door; so either way I’ll get into the garden, and I don’t care which happens!”

Lewis Carroll, Alice’s Adventures in Wonderland, Ch 1.
Proof by contradiction

\[
p \rightarrow F
\]
\[
\therefore \sim p
\]

\[
\begin{array}{ccc}
p & p \rightarrow F & \sim p \\
T & F & F \\
F & T & F \\
\end{array}
\]

Critical row

Restore us to yourself, O LORD, that we may be restored. Renew our days as of old—unless you have utterly rejected us, and you remain exceedingly angry with us.

Lam 5:21–22 (ESV)
<table>
<thead>
<tr>
<th>Mod Pon</th>
<th>Mod Tol</th>
<th>Generalization</th>
<th>Specialization</th>
<th>Elimination</th>
<th>Transitivity</th>
<th>Div into cases</th>
<th>Contradiction</th>
</tr>
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<tbody>
<tr>
<td>$p \rightarrow q$</td>
<td>$p \rightarrow q$</td>
<td>$p$</td>
<td>$p \wedge q$</td>
<td>$p \vee q$</td>
<td>$p \rightarrow q$</td>
<td>$p \vee q$</td>
<td>$p \rightarrow F$</td>
</tr>
<tr>
<td>$p$</td>
<td>$\sim q$</td>
<td>$\therefore p \vee q$</td>
<td>$\therefore q$</td>
<td>$\therefore \sim p$</td>
<td>$q \rightarrow r$</td>
<td>$p \rightarrow r$</td>
<td>$\therefore \sim p$</td>
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<td>$\therefore q$</td>
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<td></td>
<td>$\therefore p \rightarrow r$</td>
<td>$q \rightarrow r$</td>
<td></td>
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</tbody>
</table>
|       |           |               |               |             |             |               | ($a$) $t \rightarrow u$
|       |           |               |               |             |             |               | ($b$) $p \vee \sim q$
|       |           |               |               |             |             |               | ($c$) $p \rightarrow (u \rightarrow r)$
|       |           |               |               |             |             |               | ($d$) $q$
|       |           |               |               |             |             |               | ($e$) $\therefore t \rightarrow r$
3.9.2

(a) \( p \rightarrow t \)

(b) \( \sim (q \rightarrow t) \rightarrow w \)

(c) \( p \lor q \)

(d) \( \sim w \)

(e) \( \therefore t \)
3.9.8

(a) \( w \)
(b) \( q \rightarrow r \)
(c) \( t \rightarrow s \)
(d) \( u \rightarrow s \)
(e) \( (\sim t \land \sim u) \rightarrow \sim w \)
(f) \( (s \lor y) \rightarrow (p \rightarrow q) \)
(g) \( \sim (p \rightarrow r) \lor x \)
(h) \( \therefore x \)
3.9.9

(a) $p \rightarrow q$
(b) $x$
(c) $\sim (p \lor w) \rightarrow r$
(d) $q \rightarrow u$
(e) $x \rightarrow t$
(f) $w \rightarrow u$
(g) $r \lor s$
(h) $r \rightarrow F$
(i) $\therefore t \land s \land u$
<table>
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<td>$p \lor q$</td>
<td>$p \rightarrow q$</td>
<td>$p \lor q$</td>
<td>$p \rightarrow F$</td>
</tr>
<tr>
<td>$p$</td>
<td>$\sim q$</td>
<td>$\therefore p \lor q$</td>
<td>$\therefore \sim p$</td>
<td>$\therefore p \lor q$</td>
<td>$\therefore q$</td>
<td>$\therefore p \rightarrow r$</td>
<td>$\therefore r$</td>
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3.9.10

(a) $u \rightarrow \sim p$
(b) $(\sim p \lor q) \rightarrow (r \rightarrow s)$
(c) $u \land \sim w$
(d) $t \rightarrow s$
(e) $(\sim t \land \sim r) \rightarrow w$
(f) $\therefore s$
For next time:

Pg 119: 3.8.(3 & 5)
Pg 122: 3.9.(3-7)

Read carefully 3.(10 & 11)
Skim 3.(12 & 13)

Take quiz