Chapter 7 outline:
- Introduction, function equality, and anonymous functions (last Friday)
- Image and inverse images (Monday)
- Function properties, composition, and applications to programming (Wednesday)
- Cardinality (Today)
- Practice quiz and Countability (next week Monday)
- Review (Monday, Nov 29)
- Test 3, on Ch 6 & 7 (Wednesday, Dec 1)

Today:
- Homework hints
- Formal definition of cardinality
- If $A \cap B = \emptyset$, then $|A \cup B| = |A| + |B|$
- If $f : A \to B$ is one-to-one, then $|A| \leq |B|$.
**Ex. 7.6.3.** If $A, B \subseteq X$ and $f$ is one-to-one, then $F(A - B) \subseteq F(A) - F(B)$.

**Ex. 7.8.1.** If $f : A \rightarrow B$, then $f \circ i_A = f$. 
Not a function  Not a function  A function but not one-to-one or onto

Onto, not one-to-one  One-to-one, not onto  One-to-one correspondence
Onto, not one-to-one
\[ |X| \geq |Y| \]

One-to-one, not onto
\[ |X| \leq |Y| \]

One-to-one correspondence
\[ |X| = |Y| \]
Two finite sets $X$ and $Y$ have the *the same cardinality* as each other if there exists a one-to-one correspondence from $X$ to $Y$.

To use this *analytically*:
Suppose $X$ and $Y$ have the same cardinality. Then let $f$ be a one-to-one correspondence from $X$ to $Y$.
$f$ is both onto and one-to-one.

To use this *synthetically*:
*Given sets $X$ and $Y$ . . .*

[Define $f$] Let $f : X \to Y$ be a function defined as . . .
Suppose $y \in Y$. *Somehow find $x \in X$ such that $f(x) = y$. Hence $f$ is onto.*
Suppose $x_1, x_2 \in X$ such that $f(x_1) = f(x_2)$. *Somehow show $x_1 = x_2$. Hence $f$ is one-to-one.*
Since $f$ is a one-to-one correspondence, $X$ and $Y$ have the same cardinality as each other.
A finite set $X$ has cardinality $n \in \mathbb{N}$, which we write as $|X| = n$, if there exists a one-to-one correspondence from \{1, 2, \ldots n\} to $X$. Moreover, $|\emptyset| = 0$. 

\[
\begin{array}{c}
1 & \rightarrow & \text{Bob} \\
2 & \rightarrow & \text{Annabelle} \\
3 & \rightarrow & \text{Louisa} \\
4 & \rightarrow & \text{Joe} \\
5 & \rightarrow & \text{Buster} \\
\end{array}
\]
Theorem 7.12. If $A$ and $B$ are finite, disjoint sets, then $|A \cup B| = |A| + |B|$.

Theorem 7.13. If $A$ and $B$ are finite sets and $f : A \rightarrow B$ is one-to-one, then $|A| \leq |B|$.

Exercise 7.9.5. If $A$ and $B$ are finite sets and $f : A \rightarrow B$ is onto, then $|A| \geq |B|$.
\[ A \cap B = \emptyset \quad \rightarrow \quad |A \cup B| = |A| + |B| \]

\[
\begin{align*}
|A \cup B| &= |\{a_1, a_2, a_3, x, b_1, b_2\}| = 6 \\
|A| + |B| &= |\{a_1, a_2, a_3, x\}| + |\{x, b_1, b_2\}| \\
&= 4 + 3 = 7
\end{align*}
\]

\[
\begin{align*}
|A \cup B| &= |\{a_1, a_2, a_3, b_1, b_2\}| = 5 \\
|A| + |B| &= |\{a_1, a_2, a_3\}| + |\{b_1, b_2\}| \\
&= 3 + 2 = 5
\end{align*}
\]
$A \cap B = \emptyset \rightarrow |A \cup B| = |A| + |B|$
\[ A \cap B = \emptyset \quad \rightarrow \quad |A \cup B| = |A| + |B| \]

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<th>( f )</th>
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<th>( g )</th>
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<td>1</td>
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<tr>
<td>2</td>
<td>Yelemis</td>
<td>2</td>
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<tr>
<td>3</td>
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<td>Ursula</td>
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<td></td>
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<td>4</td>
<td>Tassie</td>
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<th>( x )</th>
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<td>3</td>
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</table>
\[ A \cap B = \emptyset \quad \Rightarrow \quad |A \cup B| = |A| + |B| \]
$f : A \to B$ is one-to-one $\Rightarrow |A| \leq |B|$
$f : A \to B$ is one-to-one $\implies |A| \leq |B|$
For next time:

Pg 359: 7.9.(1 & 2)