Semester roadmap:

Ch 1 & 2: Raw materials
Ch 3: Formal logic
— Test 1, Feb 12 —
Ch 4: Proofs
Ch 5: Relations
— Test 2, Mar 20 —
Ch 6: Self reference
Ch 7: Functions
— Test 3, Apr 19 —

Chapter 3 roadmap:

Today: Logical equivalences (Game 1)
Wednesday: Conditionals (SML)
Friday: Arguments (Game 2)
Next week Monday: Predicates and quantification (SML)
Next week Wednesday: Quantified arguments (Game 3)
Next week Friday: Review for test
Chapter 3 outline:

▶ Propositions, boolean logic, logical equivalences. **Game 1** (Monday)
▶ Conditional propositions. **SML** (Today)
▶ Arguments. **Game 2** (Friday)
▶ Predicates and quantification. **SML** (next week Monday)
▶ Quantified arguments. **Game 3** (next week Wednesday)
▶ Review for test (next week Friday)
So far:

- \( \mathbb{B} = \{ T, F \} \), \( \land \), \( \lor \), \( \sim \), propositional calculus
- Verifying logical equivalences between propositional forms (Game 1)

Today—how to model propositional forms that have an if/then structure (§3.(5–7)):

- Highlight the most important parts
- Highlight the most confusing parts
- Work on some SML examples
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<th>$p$</th>
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<th>$p \land q$</th>
<th>$p \lor q$</th>
<th>$\sim p$</th>
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If $12$ divides $36$ evenly, then $3$ divides $72$ evenly.

If $3 < 72$, then $3$ divides $72$ evenly.

If $12$ divides $36$ evenly, then $72 < 3$.

If $72 < 3$, then $3$ divides $72$ evenly.

If $72 < 3$, then $12$ divides $3$ evenly.
1. Bob passed through $P$.
2. Bob passed through $N$.
3. Bob passed through $M$.
4. If Bob passed through $O$, then Bob passed through $F$.
5. If Bob passed through $K$, then Bob passed through $L$.
6. If Bob passed through $L$, then Bob passed through $K$.

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Based on example by Susanna Epp, 2006
“If Fred was at the dock at midnight, then he’s the murderer.”

“If it’s raining at home and the windows are still open, then water is coming in.”

“If I were John and John were me, then he’d be six and I’d be three.” — A. A. Milne

“If the dryer is finished, then unload it.”

“If you finish your spinach, then I will give you some cake.”

“If it rains tomorrow, the zucchini will sprout.”
An even degree is a **necessary condition** for a polynomial to have no real roots.  
*means*  
If a polynomial function has no real roots, then it has an even degree.

A positive global minimum is a **sufficient condition** for a polynomial to have no real roots.  
*means*  
If a polynomial function has a positive global minimum, then it has no real roots.

Values all of the same sign is a **necessary** and **sufficient** condition for a polynomial to have no real roots.  
*means*  
A polynomial function has values all of the same sign if and only if the function has no real roots.
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<th>$\sim q \rightarrow \sim p$</th>
<th>$p \land \sim q$</th>
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With respect to the conditional proposition

*If the jar is open, then the cookies are gone.*

identify each of the following propositions.

- The jar is open.
- The cookies are gone.
- If the cookies are gone, then the jar is open.
- If the jar is not open, then the cookies are not gone.
- If the cookies are not gone, then the jar is not open.
Conditional expression:

if (expr1) then (expr2) else (expr3)
For next time:
   
Pg 108: 3.5.(1 & 2)
Pg 114: 3.7.(1, 2, 7, 8, 9, 12, 13)

Read 3.(8 & 9)

Take quiz