Chapter 4 roadmap:
▶ Subset proofs (last week Wednesday)
▶ Set equality and emptiness proofs (last week Friday)
▶ Conditional and biconditional proofs (Today)
▶ Proofs about powersets (Wednesday)
▶ From theorems to algorithms (Friday)
▶ (Start Chapter 5 week after next)

Today:
▶ Proofs of conditional propositions
▶ Proofs about numbers
▶ Proofs of biconditional propositions

General forms:

1. **Facts** ($p$)
   
   Set forms
   
   1. Subset $X \subseteq Y$
   2. Set equality $X = Y$
   3. Set emptiness $X = \emptyset$

2. **Conditionals** ($p \rightarrow q$)

3. **Biconditionals** ($p \leftrightarrow q$)
Hypothetical conditional from Game 3:

To prove $p \rightarrow q$
Suppose $p$

\[ \ldots \]
\[ q \]

$p \rightarrow q$
An integer $x$ is even if $\exists \ k \in \mathbb{Z} \ | \ x = 2k$.

An integer $x$ is odd if $\exists \ k \in \mathbb{Z} \ | \ x = 2k + 1$.

“Axiom 3.” If $x, y \in \mathbb{Z}$, then $x + y \in \mathbb{Z}$. (Closure of addition)

“Axiom 4.” If $x, y \in \mathbb{Z}$, then $x \cdot y \in \mathbb{Z}$. (Closure of multiplication)

“Axiom 5.” If $x \in \mathbb{Z}$, then $x$ is even iff $x$ is not odd.

$\forall \ x, y \in \mathbb{Z}, \ x \ | \ y$ (read, “$x$ divides $y$”) if $\exists \ k \in \mathbb{Z} \ | \ x \cdot k = y$.

Note that $y/x = k$ or $\frac{y}{x} = k$ or $x \ | \ \frac{k}{y}$. 
For next time:

Pg 162: 4.5.(1, 4, 5)
Pg 164: 4.6.(2 & 5)
Pg 165: 4.7.(1 & 6)

Review 2.4, especially Ex 2.4.15
Skim 4.9

Take quiz on Schoology