Chapter 4 roadmap:
- Subset proofs (Wednesday)
- Set equality and emptiness proofs (Today)
- Conditional and biconditional proofs (next week Wednesday)
- Proofs about powersets (new week Friday)
- From theorems to algorithms (week-after Monday)
- (Start Chapter 5 week-after Wednesday)

Today:
- Proofs that sets are equal
- Proofs that a set is empty

General forms:

1. Facts ($p$)
   
   **Set forms**
   
   1. Subset $X \subseteq Y$
   2. Set equality $X = Y$
   3. Set emptiness $X = \emptyset$

2. Conditionals ($p \rightarrow q$)

3. Biconditionals ($p \leftrightarrow q$)
\[ A \times (B - C) \subseteq (A \times B) - (A \times C). \]

**Proof (long version).** Suppose \( x \in A \times (B - C). \) By definition of Cartesian product, \( x = (a, d) \) for some \( a \in A \) and \( d \in B - C. \) By definition of difference, \( d \in B \) and \( d \notin C. \)

By definition of Cartesian product, \((a, d) \in A \times B. \) Also by definition of Cartesian product, this time used negatively, \((a, d) \notin A \times C. \)

[That is, we rewrite \( d \notin C \) as \( \sim (d \in C). \) By generalization, \( \sim (d \in C \land a \in A). \) By definition of Cartesian product, \( \sim ((a, d) \in A \times C). \) This can be rewritten as \((a, d) \notin A \times C. \)]

By definition of difference, \((a, d) \in (A \times B) - (A \times C). \) By substitution, \( x \in (A \times B) - (A \times C). \) Therefore, by definition of subset, \( A \times (B - C) \subseteq (A \times B) - (A \times C). \) □
\[ A \times (B - C) \subseteq (A \times B) - (A \times C). \]

**Proof (short version).** Suppose \((a, d) \in A \times (B - C)\). By definition of Cartesian product, \(a \in A\) and \(d \in B - C\).

By definition of difference, \(d \in B\) and \(d \notin C\). By definition of Cartesian product, \((a, d) \in A \times B\) and \((a, d) \notin A \times C\).

By definition of difference, \((a, d) \in (A \times B) - (A \times C)\). Therefore, by definition of subset, \(A \times (B - C) \subseteq (A \times B) - (A \times C)\). \(\square\)
For next time:

Pg 160: 4.3.(3, 14, 15, 18)
Pg 161: 4.4.(5 & 6)

See assignment on Canvas for hint on Ex 4.3.15.

Read 4.(5–8)
Take quiz