Chapter 4 roadmap:

- Subset proofs (Monday)
- Set equality and emptiness proofs (Today)
- Conditional and biconditional proofs (Friday)
- Proofs about powersets (new week Wednesday)
- From theorems to algorithms (next week Friday)
- (Start Chapter 5 week after next)

Today:
- Proofs that sets are equal
- Proofs that a set is empty

General forms:

1. **Facts** \((p)\)
   
   **Set forms**
   
   1. Subset \(X \subseteq Y\)
   2. Set equality \(X = Y\)
   3. Set emptiness \(X = \emptyset\)

2. **Conditionals** \((p \rightarrow q)\)

3. **Biconditionals** \((p \leftrightarrow q)\)
\[ A \times (B - C) \subseteq (A \times B) - (A \times C). \]

**Proof (long version).** Suppose \( x \in A \times (B - C) \). By definition of Cartesian product, \( x = (a, d) \) for some \( a \in A \) and \( d \in B - C \). By definition of difference, \( d \in B \) and \( d \notin C \).

By definition of Cartesian product, \((a, d) \in A \times B\). Also by definition of Cartesian product, this time used negatively, \((a, d) \notin A \times C\).

[That is, we rewrite \( d \notin C \) as \( \sim (d \in C) \). By generalization, \( \sim (d \in C \land a \in A) \). By definition of Cartesian product, \( \sim ((a, d) \in A \times C) \). This can be rewritten as \((a, d) \notin A \times C\).]

By definition of difference, \((a, d) \in (A \times B) - (A \times C)\). By substitution, \( x \in (A \times B) - (A \times C) \). Therefore, by definition of subset, \( A \times (B - C) \subseteq (A \times B) - (A \times C) \). \( \square \)
\[ A \times (B - C) \subseteq (A \times B) - (A \times C). \]

**Proof (short version).** Suppose \((a, d) \in A \times (B - C).\) By definition of Cartesian product, \(a \in A\) and \(d \in B - C.\)

By definition of difference, \(d \in B\) and \(d \notin C.\) By definition of Cartesian product, \((a, d) \in A \times B\) and \((a, d) \notin A \times C.\)

By definition of difference, \((a, d) \in (A \times B) - (A \times C).\) Therefore, by definition of subset, \(A \times (B - C) \subseteq (A \times B) - (A \times C).\) \(\square\)
For next time:

Pg 160: 4.3.(3, 14, 15, 18)
Pg 161: 4.4.(5 & 6)

See assignment on Schoology for hint on Ex 4.3.15.

Read 4.(5–8)