“Stand-alone topics” outline:

▶ Foldl, and how to model mathematical functions (Last Friday)
▶ Fixed-point iteration (today)
▶ The Huffman encoding (Wednesday)
▶ Review for the final exam (Friday)

Today:

▶ Currying functions
▶ Specific problem: calculating square roots
▶ Medium-general problem: finding roots of real-valued functions
▶ Very general solution: Fixed-point iteration
▶ Applying general solution to specific problems
arity, n  The number of elements by virtue of which something is unary, binary, etc.

curry, v.1  (a) To rub down or dress (a horse etc.) with a comb.  (b) (Applied to persons) to seek to win favor, or ingratiate oneself with another.

curry, v.2  Obsolete.  To ride or run with haste or rapidity

curry, v.3  To flavor or prepare with curry or curry-powder.

curry, v.4  To transform a function that takes multiple arguments into a function that takes a single argument and returns a new function that takes the remainder of the arguments and returns the result.

Definition of arity and the first three definitions of curry are adapted from the Oxford English Dictionary. The fourth definition of curry is adapted from wiktionary.org.
\[ f(x) = x^2 - c \]

\[ g(x) \]

\[ f(x)_{i+1} = f(x_i) \]

\[ x, x' \]

\[ (x_i, f(x_i)) \]

\[ (x_{i+1}, f(x_{i+1})) \]

\[ (0, -\delta) \]

\[ (-\sqrt{c}, 0) \]

\[ (\sqrt{c}, 0) \]
Point-slope form: Given a point \((x_1, y_1)\) and a slope \(m\),

\[
\begin{align*}
  y - y_1 &= m \cdot (x - x_1) \\
  y &= m \cdot (x - x_1) + y_1
\end{align*}
\]

To find root of function \(f\) near a guess \(x_i\), draw a tangent line at \((x_1, f(x_1))\):

\[g(x) = f'(x_1) \cdot (x - x_1) + f(x_1)\]

Then find where this line hits the \(x\)-axis:

\[0 = f'(x_1)(x - x_1) + f(x_1)\]

\[x = \frac{x_1f''(x_1) - f(x_1)}{f'(x_1)}\]

\[= x_1 - \frac{f(x_1)}{f'(x_1)}\]

Consider that our next guess. In general,

\[x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}\]
In general, to find a new, improved guess $x_{i+1}$ for a root of function $f$ from current guess $x_i$,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

To find $\sqrt{c}$, apply this to $f(x) = x^2 - c$.

$$f'(x) = 2x$$

And so the guess-improving function is

$$I(x) = x - \frac{x^2 - c}{2x}$$

Algorithm for finding an approximation for a root:

*If our guess is good enough, then stop.*

*Otherwise, improve it and try again.*
**Fixed-point iteration.** Algorithm for any iterative guess-improving process:

*If our guess is good enough, then stop.*

*Otherwise, improve it and try again.*

**Elements:**
- Guess-improver
- Current guess
- Good-enough tester

For finding roots using Newton’s method, the guess-improver is

\[ G(x) = x - \frac{f(x)}{f'(x)} \]

What if \( x \) is an exact root, that is, if \( f(x) = 0 \)?
Lessons:

- Interaction between mathematics and computer science: An algorithm solves a mathematical problem.
- Interaction between continuous and discrete mathematics: The *continuous* functions are themselves *discrete* objects.
- Generalization in math and computer science: Functions and algorithms can be parameterized to become more general.
For next time:

Pg 375: 7.13.(1,3)

Skim 6.12, new version found on Schoology