Chapter 1 & 2 outline:

▶ Introduction, sets and elements (last week Monday)
▶ Set operations; visual verification of set propositions (last week Wednesday)
▶ Introduction to SML; cardinality and Cartesian products (last week Friday)
▶ Making types in SML (this week Wednesday)
▶ Functions in SML (last week Friday)
▶ Functions on lists (Today)
▶ Powersets; a language processor (Friday)
▶ (Begin chapter 3, Propositions, next week Monday)

Today:

▶ Review of lists
▶ Type analysis of lists
▶ Functions on lists
▶ (Time permitting) Begin powersets
[t1([5, 12, 6])@[8, 9]]
hd([12, 5, 6])::[2, 7]
[[[(2.3, 5), (8.1, 6)], []]]
([1, 12, 81], ["a", "bc"])

Powersets

- **Informal definition:** The powerset of a set is the set of all subsets of that set.
- **Formal definition:** The powerset of a set $X$ is

$$\mathcal{P}(X) = \{ Y \mid Y \subseteq X\}$$

- For “set of sets,” think “box of boxes.”
- **Examples:**
Why powersets seem to throw some people:

▶ The elements of a powerset are themselves sets.
▶ Suppose \( X \subseteq \mathcal{U} \). Then

▶ If \( x \in X \), then \( x \in \mathcal{U} \)
▶ \( \mathcal{P}(X) \nsubseteq \mathcal{U} \), but rather \( \mathcal{P}(X) \subseteq \mathcal{P}(\mathcal{U}) \)
▶ If \( A \in \mathcal{P}(X) \), then \( A \in \mathcal{P}(\mathcal{U}) \)

▶ \( \mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset \). \( |\emptyset| = 0 \), but \( |\{\emptyset\}| = 1 \)
For next time:

If you had trouble on the programming problems from last time, ask for help and try again.
Pg 70: 2.1.(2-4, 9, 10) [on paper]
Pg 74: 2.2.(2, 3, 8, 9, 11) [through turn-in page]

See notes on Ex 2.2.8 and 2.2.9 on the Canvas description of the assignment for clarifications and hints. See also the code from class for “starter code.” You do not need to include your SML code with your on-paper problems that you turn in.

Read 2.(4 & 5)
Take quiz
(There will be a follow-up quiz after class Friday)