Where we are:

- Making types in SML (last week Wednesday)
- Functions in SML (last week Friday)
- Lists and functions on lists (Wednesday)
- Powersets; a language processor (Today)
- Propositional forms, logical equivalence [Start Chapter 3] (Friday)

Today:

- Powersets
  - Definition
  - Exploration
- A language processor
  - Case expressions and option types
  - The language processor itself
  - Introducing the semester project
Review

- List literals: [1, 4, 12, 3], []
- Analytic operations: hd, tl
- Synthetic operations: :: (cons), @ (cat)
- Lists vs tuples
- Type analysis problems:

\[
\text{t1}([5, 12, 6]) \odot [8, 9]
\]

- Lists as models for sets
Powersets

▶ Informal definition: The powerset of a set is the set of all subsets of that set.
▶ Formal definition: The powerset of a set $X$ is

$$\mathcal{P}(X) = \{ Y \mid Y \subseteq X \}$$

▶ For “set of sets,” think “box of boxes.”
▶ Examples:
Why powersets seem to throw some people:

- The elements of a powerset are themselves sets.
- Suppose $X \subseteq U$. Then
  - If $x \in X$, then $x \in U$
  - $P(X) \not\subseteq U$, but rather $P(X) \subseteq P(U)$
  - If $A \in P(X)$, then $A \in P(U)$

- $P(\emptyset) = \{\emptyset\} \neq \emptyset$. $|\emptyset| = 0$, but $|\{\emptyset\}| = 1$
Which are true?

\[ \{3\} \in \mathcal{P}(\{1, 2, 3, 4, 5\}) \quad 3 \in \mathcal{P}(\{1, 2, 3, 4, 5\}) \]

\[ \{3\} \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\}) \quad 3 \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\}) \]

\[ a \in A \text{ iff } \{a\} \in \mathcal{P}(A) \quad a \in A \text{ iff } \{a\} \subseteq \mathcal{P}(A) \]

\[ A \subseteq B \text{ iff } A \subseteq \mathcal{P}(B) \quad A \subseteq B \text{ iff } A \in \mathcal{P}(B) \]
Which are true?

\{ A \} \subseteq \mathcal{P}(A) \quad A \in \mathcal{P}(A)

\{ A \} \in \mathcal{P}(A) \quad \mathbb{Z} \in \mathcal{P}(\mathbb{R})

\emptyset \in \mathcal{P}(A) \quad \emptyset = \mathcal{P}(\emptyset)
Note that

- $a \in A$ iff \{a\} $\in \mathcal{P}(A)$
- $A \subseteq B$ iff $A \in \mathcal{P}(B)$
- $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$
- $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$
Observe

\[ \mathcal{P}(\{1, 2, 3\}) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\} \} = \{ \emptyset, \{1\}, \{1, 2\}, \{1, 3\}, \{2\}, \{3\}, \{2, 3\}, \{1, 2, 3\} \} \]

\[ = \mathcal{P}(\{2, 3\}) \cup \left[ \text{1 added to each set of } \mathcal{P}(\{2, 3\}) \right] = \mathcal{P}(\{2, 3\}) \cup \{ \{1\} \cup X \mid X \in \mathcal{P}(\{2, 3\}) \} \]

If \( a \in A \), then \( \mathcal{P}(A) = \mathcal{P}(A - \{a\}) \cup \{ \{a\} \cup X \mid X \in \mathcal{P}(A - \{a\}) \} \)
What is $|\mathcal{P}(X)|$ in terms of $|X|$?
Grammar:

Sentence $\rightarrow$ NounPhrase Predicate PrepPhrase$_{opt}$

NounPhrase $\rightarrow$ Article Adjective$_{opt}$ Noun

Predicate $\rightarrow$ Adverb$_{opt}$ VerbPhrase
Grammar, continued:

\[
\text{VerbPhrase} \rightarrow \begin{cases} 
\text{TransitiveVerb NounPhrase} \\
\text{IntransitiveVerb} \\
\text{LinkingVerb Adjective}
\end{cases}
\]

\[
\text{PrepPhrase} \rightarrow \text{Preposition NounPhrase}
\]
Vocabulary:

Articles: a the

Adjectives: big bright fast beautiful smart red smelly

Nouns: man woman dog unicorn ball field flea tree

Adverbs: quickly slowly happily dreamily

Transitive verbs: chased saw greeted bit loved

Intransitive verbs: ran slept sang

Linking verbs: was felt seemed

Prepositions: in on through with
For next time:

Take “follow-up quiz”

Pg 74: 2.2.(13, 15)
Pg 82: 2.4.(8-12, 14 & 15)

Read 3.(1-4)
Take quiz