Where we are:

- Making types in SML (last week Wednesday)
- Functions in SML (last week Friday)
- Lists (Monday)
- Functions on lists (Wednesday)
- Powersets; a language processor (**Today**)
- Propositional forms, logical equivalence [Start Chapter 3] (next week Monday)

Today:

- A couple more list examples
- Powersets
  - Definition
  - Exploration
- A language processor
  - Case expressions and option types
  - The language processor itself
  - Introducing the semester project
Review

- List literals: [1, 4, 12, 3], []
- Analytic operations: hd, tl
- Synthetic operations: :: (cons), @ (cat)
- Lists vs tuples
- Type analysis problems:

\[
[\text{tl}([5, 12, 6]) @ [8, 9]]
\]

- Lists as models for sets
Powersets

- Informal definition: The powerset of a set is the set of all subsets of that set.
- Formal definition: The powerset of a set $X$ is
  \[ \mathcal{P}(X) = \{ Y \mid Y \subseteq X \} \]
- For “set of sets,” think “box of boxes.”
- Examples:
Why powersets seem to throw some people:

- The elements of a powerset are themselves sets.
- Suppose $X \subseteq U$. Then
  - If $x \in X$, then $x \in U$
  - $\mathcal{P}(X) \not\subseteq U$, but rather $\mathcal{P}(X) \subseteq \mathcal{P}(U)$
  - If $A \in \mathcal{P}(X)$, then $A \in \mathcal{P}(U)$
- $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$. $|\emptyset| = 0$, but $|\{\emptyset\}| = 1$
Which are true?

\( \{3\} \in \mathcal{P}(\{1, 2, 3, 4, 5\}) \quad 3 \in \mathcal{P}(\{1, 2, 3, 4, 5\}) \)

\( \{3\} \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\}) \quad 3 \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\}) \)

\( a \in A \iff \{a\} \in \mathcal{P}(A) \quad a \in A \iff \{a\} \subseteq \mathcal{P}(A) \)

\( A \subseteq B \iff A \subseteq \mathcal{P}(B) \quad A \subseteq B \iff A \in \mathcal{P}(B) \)
Which are true?

\{A\} \subseteq \mathcal{P}(A)

A \in \mathcal{P}(A)

\{A\} \in \mathcal{P}(A)

\mathbb{Z} \in \mathcal{P}(\mathbb{R})

\emptyset \in \mathcal{P}(A)

\emptyset = \mathcal{P}(\emptyset)
Note that

- $a \in A$ iff $\{a\} \in \mathcal{P}(A)$
- $A \subseteq B$ iff $A \in \mathcal{P}(B)$
- $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$
- $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$
Observe

\[ P(\{1, 2, 3\}) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\} \} \]

\[ = P(\{2, 3\}) \cup \left[ \{1\} \text{ added to each set of } P(\{2, 3\}) \right] \]

\[ = P(\{2, 3\}) \cup \{ \{1\} \cup X \mid X \in P(\{2, 3\}) \} \]

If \( a \in A \), then \( P(A) = P(A - \{a\}) \cup \{ \{a\} \cup X \mid X \in P(A - \{a\}) \} \)
What is $|\mathcal{P}(X)|$ in terms of $|X|$?
Grammar:

\[
\text{Sentence} \rightarrow \text{NounPhrase} \text{ Predicate} \text{ PrepPhrase}_{\text{opt}}
\]

\[
\text{NounPhrase} \rightarrow \text{Article} \text{ Adjective}_{\text{opt}} \text{ Noun}
\]

\[
\text{Predicate} \rightarrow \text{Adverb}_{\text{opt}} \text{ VerbPhrase}
\]
Grammar, continued:

\[
\text{VerbPhrase} \rightarrow \begin{cases} \\
\text{TransitiveVerb NounPhrase} \\
\text{IntransitiveVerb} \\
\text{LinkingVerb Adjective} \end{cases}
\]

\[
\text{PrepPhrase} \rightarrow \text{Preposition NounPhrase}
\]
Vocabulary:

Articles: a the

Adjectives: big bright fast beautiful smart red smelly

Nouns: man woman dog unicorn ball field flea tree

Adverbs: quickly slowly happily dreamily

Transitive verbs: chased saw greeted bit loved

Intransitive verbs: ran slept sang

Linking verbs: was felt seemed

Prepositions: in on through with
For next time:

Take “follow-up quiz”

Pg 74: 2.2.(13, 15)
Pg 82: 2.4.(8-12, 14 & 15)

Read 3.(1-4)
Take quiz