Where we are:

- Making types in SML (last week Wednesday)
- Functions in SML (last week Friday)
- Lists and functions on lists (Wednesday)
- Powersets; a language processor (Today)
- Propositional forms, logical equivalence [Start Chapter 3] (next week Monday)

Today:

- Powersets
  - Definition
  - Exploration
- A language processor
  - Case expressions and option types
  - The language processor itself
  - Introducing the semester project
Review

- List literals: [1, 4, 12, 3], []
- Analytic operations: hd, tl
- Synthetic operations: :: (cons), @ (cat)
- Lists vs tuples
- Type analysis problems:

\[
\begin{align*}
[ & \text{tl}([5, 12, 6]) \@ [8, 9] ] \\
[ & \text{int list} \@ \text{int list} ] \\
[ & \text{int list} \@ \text{int list} ] \\
[ & \text{int list} \@ \text{int list list} ]
\end{align*}
\]

- Lists as models for sets
Powersets

- Informal definition: The powerset of a set is the set of all subsets of that set.
- Formal definition: The powerset of a set $X$ is
  \[ \mathcal{P}(X) = \{ Y \mid Y \subseteq X \} \]
- For “set of sets,” think “box of boxes.”
- Examples:
Why powersets seem to throw some people:

▶ The elements of a powerset are themselves sets.
▶ Suppose $X \subseteq U$. Then
  
  ▶ If $x \in X$, then $x \in U$
  
  ▶ $\mathcal{P}(X) \not\subseteq U$, but rather $\mathcal{P}(X) \subseteq \mathcal{P}(U)$
  
  ▶ If $A \in \mathcal{P}(X)$, then $A \in \mathcal{P}(U)$
  
  ▶ $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$. $|\emptyset| = 0$, but $|\{\emptyset\}| = 1$
Which are true?

\{3\} \in \mathcal{P}(\{1, 2, 3, 4, 5\}) 
\quad 3 \in \mathcal{P}(\{1, 2, 3, 4, 5\})

\{3\} \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\}) 
\quad 3 \subseteq \mathcal{P}(\{1, 2, 3, 4, 5\})

a \in A \iff \{a\} \in \mathcal{P}(A) 
\quad a \in A \iff \{a\} \subseteq \mathcal{P}(A)

A \subseteq B \quad \text{iff} \quad A \subseteq \mathcal{P}(B) 
A \subseteq B \quad \text{iff} \quad A \in \mathcal{P}(B)
Which are true?

\[ A \subseteq B \text{ iff } A \subseteq \mathcal{P}(B) \]

\[ \{A\} \subseteq \mathcal{P}(A) \]

\[ A \in \mathcal{P}(A) \]

\[ \{A\} \in \mathcal{P}(A) \]

\[ \mathbb{Z} \in \mathcal{P}(\mathbb{R}) \]

\[ \emptyset = \mathcal{P} (\emptyset) \]
Note that

- $a \in A$ iff $\{a\} \in \mathcal{P}(A)$

- $A \subseteq B$ iff $A \in \mathcal{P}(B)$

- $A \subseteq B$ iff $\mathcal{P}(A) \subseteq \mathcal{P}(B)$

- $\mathcal{P}(\emptyset) = \{\emptyset\} \neq \emptyset$
Observe

\[ \mathcal{P}({1, 2, 3}) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\} = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{2, 3\} \} \]

\[ = \mathcal{P}({2, 3}) \cup \left[ \text{1 added to each set of } \mathcal{P}({2, 3}) \right] = \mathcal{P}({2, 3}) \cup \left\{ \{1\} \cup X \mid X \in \mathcal{P}({2, 3}) \right\} \]

If \( a \in A \), then \( \mathcal{P}(A) = \mathcal{P}(A - \{a\}) \cup \{ \{a\} \cup X \mid X \in \mathcal{P}(A - \{a\}) \} \)
What is $|\mathcal{P}(X)|$ in terms of $|X|$?
Grammar:

\[ \text{Sentence} \rightarrow \text{NounPhrase} \text{ Predicate} \text{ PrepPhrase}_{\text{opt}} \]

\[ \text{NounPhrase} \rightarrow \text{Article} \text{ Adjective}_{\text{opt}} \text{ Noun} \]

\[ \text{Predicate} \rightarrow \text{Adverb}_{\text{opt}} \text{ VerbPhrase} \]
Grammar, continued:

\[
\text{VerbPhrase} \rightarrow \begin{cases} \\
\text{TransitiveVerb NounPhrase} \\
\text{IntransitiveVerb} \\
\text{LinkingVerb Adjective} \\
\end{cases} \\
\text{PrepPhrase} \rightarrow \text{Preposition NounPhrase}
\]
**Vocabulary:**

**Articles:** a the

**Adjectives:** big bright fast beautiful smart red smelly

**Nouns:** man woman dog unicorn ball field flea tree

**Adverbs:** quickly slowly happily dreamily

**Transitive verbs:** chased saw greeted bit loved

**Intransitive verbs:** ran slept sang

**Linking verbs:** was felt seemed

**Prepositions:** in on through with
For next time:

If you had trouble on the programming problems from last time, ask for help and try again.

Pg 74: 2.2.(11, 13, 15)
Pg 82: 2.4.(8-12, 14 & 15)
Read 3.(1-4)