Chapter 4 roadmap:
▶ Subset proofs (Last week Wednesday)
▶ Set equality and emptiness proofs (Last week Friday)
▶ Conditional and biconditional proofs (Monday)
▶ Proofs about powersets (Today)
▶ From theorems to algorithms (Friday)
▶ (Start Chapter 5 relations next week)

Today: Case study of large proof (powersets)
▶ Review of powersets and their recursive structure
▶ Big result
▶ Warm-up proofs
▶ Proving the big result
Consider the set $A = \{1, 2, 3, 4, 5\}$. Which of the following is true about the powerset $\mathcal{P}(A)$? (Only one is true.)

\[
\begin{align*}
\{3\} & \in \mathcal{P}(A) & 3 & \in \mathcal{P}(A) \\
\{3\} & \subseteq \mathcal{P}(A) & 3 & \subseteq \mathcal{P}(A)
\end{align*}
\]
\[ A = \{a, b, c\} \quad \mathcal{P}(A) = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}, \{b, c\}, \{b\}, \{c\}, \emptyset\} \]

\[ A - \{a\} = \{b, c\} \quad \mathcal{P}(A - \{a\}) = \{\{b, c\}, \{b\}, \{c\}, \emptyset\} \]

\[ \{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}\} \]

\[ \mathcal{P}(A) = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}\} = \{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} \cup \mathcal{P}(A - \{a\}) \]
\[ A = \{a, b, c\} \quad \mathcal{P}(A - \{a\}) = \{\{b, c\}, \{b\}, \{c\}, \emptyset\} \]

\[
\{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}\}
\]
If $a \in A$, then $\mathcal{P}(A)$ consists in $\mathcal{P}(A - \{a\})$ and $\{ C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\}) \}$

**Corollary 4.12.** If $a \in A$, then $\mathcal{P}(A - \{a\})$ and $\{ C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\}) \}$ make a partition of $\mathcal{P}(A)$. 
$A \subseteq B$ iff $A \in \mathcal{P}(B)$

$A \in \mathcal{P}(A)$

$\emptyset \in \mathcal{P}(A)$

$a \in A$ iff $\{a\} \in \mathcal{P}(A)$
Warm-up proofs:

**Theorem 4.7.** If $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, then $A \subseteq B$.

**Exercise 4.9.1.** If $B \subseteq A$, then $\mathcal{P}(B) - \mathcal{P}(A) = \emptyset$. 
Roadmap

Corollary 4.12
\( \mathcal{P}(A - \{a\}) \) and \( \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} \)
make a partition of \( \mathcal{P}(A) \).

\[ \uparrow \quad \downarrow \]

Theorem 4.11 / Exercise 4.9.6
\( \mathcal{P}(A - \{a\}) \cap \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} = \emptyset \)

\[ \uparrow \quad \downarrow \]

Theorem 4.10.
\( \mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} = \mathcal{P}(A) \)

\[ \uparrow \quad \downarrow \]

Lemma 4.9.
\( \mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} \subseteq \mathcal{P}(A) \)

\[ \uparrow \quad \downarrow \]

Lemma 4.8.
\( \mathcal{P}(A) \subseteq \mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} \)
For next time:

*Pg 174: 4.9.(1, 3, 4, 6)*

*Skim 4.(10 & 11)*