Chapter 4 roadmap:

- Subset proofs (last week Wednesday)
- Set equality and emptiness proofs (last week Friday)
- Conditional and biconditional proofs (Monday)
- Proofs about powersets (Today)
- From theorems to algorithms (Friday)
- (Start Chapter 5 next week)

Today: Case study of large proof (powersets)

- Review of powersets and their recursive structure
- Big result
- Warm-up proofs
- Proving the big result
Which are true?

\[ \mathcal{P}(\emptyset) = \emptyset \quad \quad \mathcal{P}(\emptyset) = \{\emptyset\} \]
\[ \mathcal{P}(\emptyset) = \{\emptyset\} \quad \quad \mathcal{P}(\emptyset) = \{\emptyset, \emptyset\} \]
\[ \mathcal{P}(\{1\}) = \{1\} \quad \quad \mathcal{P}(\{1\}) = \{\{1\}\} \]
\[ \mathcal{P}(\{1\}) = \{\emptyset, \{1\}\} \]
\[ A \in \mathcal{P}(A) \quad \quad A \subseteq \mathcal{P}(A) \]
\[
A = \{a, b, c\} \quad \mathcal{P}(A) = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}, \\
\{b, c\}, \{b\}, \{c\}, \emptyset\}
\]

\[
A - \{a\} = \{b, c\} \quad \mathcal{P}(A - \{a\}) = \{\{b, c\}, \{b\}, \{c\}, \emptyset\}
\]

\[
\{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}\}
\]

\[
\mathcal{P}(A) = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}, \\
\{b, c\}, \{b\}, \{c\}, \emptyset\} = \{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} \\
\cup \mathcal{P}(A - \{a\})
\]
\[ A = \{a, b, c\} \quad \mathcal{P}(A - \{a\}) = \{\{b, c\}, \{b\}, \{c\}, \emptyset\} \]

\[ \{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}\} \]
If \( a \in A \), then \( \mathcal{P}(A) \) consists in \( \mathcal{P}(A \setminus \{a\}) \) and \( \{ C \cup \{a\} \mid C \in \mathcal{P}(A \setminus \{a\}) \} \).

**Corollary 4.12.** If \( a \in A \), then \( \mathcal{P}(A \setminus \{a\}) \) and \( \{ C \cup \{a\} \mid C \in \mathcal{P}(A \setminus \{a\}) \} \) make a partition of \( \mathcal{P}(A) \).
$A \subseteq B \iff A \in \mathcal{P}(B)$

$A \in \mathcal{P}(A)$

$\emptyset \in \mathcal{P}(A)$

$a \in A \iff \{a\} \in \mathcal{P}(A)$
Warm-up proofs:

**Theorem 4.7.** If $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, then $A \subseteq B$.

**Exercise 4.9.1.** If $B \subseteq A$, then $\mathcal{P}(B) - \mathcal{P}(A) = \emptyset$. 
Roadmap

**Corollary 4.12**
\[ \mathcal{P}(A - \{a\}) \text{ and } \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} \]
make a partition of \( \mathcal{P}(A) \).

\[ \mathcal{P}(A - \{a\}) \cap \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} = \emptyset \]

**Theorem 4.10.**
\[ \mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} = \mathcal{P}(A) \]

**Lemma 4.9.**
\[ \mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} \subseteq \mathcal{P}(A) \]

**Lemma 4.8.**
\[ \mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} \subseteq \mathcal{P}(A) \]
For next time:

Pg 174: 4.9.(1, 3, 4, 6)

Skim 4.(10 & 11)

Take quiz