Chapter 3 roadmap:

- Propositions, boolean logic, logical equivalences. **Game 1** (last week Monday)
- Conditional propositions. **SML** (last week Wednesday)
- Arguments. **Game 2** (last week Friday)
- Predicates and quantification. **SML** (Today)
- Quantified arguments. **Game 3** (Wednesday)
- Review for test. (Friday)
- Test 1. (Next week Monday)

Today:

- Predicates
- Quantification
- Practice quantification using programming problems

Project proposal due Friday, Oct 1.
Propositions:
- $3 < 5$
- It’s Thursday and it is snowing.
- If $3 < 5$ then $12 < 67$.

Propositional forms:
- $p \land q$
- $p \rightarrow q$
Four ways to interpret/define the idea of a *predicate*

- A predicate is a proposition with a parameter.
  \[ x < 5 \quad \text{x is orange} \]

- A predicate is a function whose value is true or false.
  \[ P(x) = x < 5 \quad Q(x) = \text{x is orange} \]

- A predicate is a part of a sentence that complements a noun phrase to make a proposition.
  A pumpkin is orange.

- A predicate is a truth set
  \[ P : \mathbb{N} \rightarrow \mathbb{B}, P(x) = x < 5 \quad Q(x) = \text{x is orange} \]
  Truth set: \{1, 2, 3, 4\} \quad \{ \text{pumpkin, fall leaves, orange juice, ...} \}
Universal quantification

“For all multiples of 3, the sum of their digits is a multiple of 3.”

Let $D$ be the set of multiples of 3, that is

$D = \{ n \in \mathbb{N} \mid n \mod 3 = 0 \} = \{3, 6, 9, 12, 15, 18, \ldots \}$

$\forall x \in D, \text{sum}(\text{digify}(x)) \in D$

Other examples:

- $\forall x \in \{5, 7, 19, 23, 43\}$, $x$ is prime.
- $\forall x \in \{4, 16, 25, 31\}$, $x$ is a perfect square.
Existential quantification

“There is a multiple of 3 that is not a perfect square.”

\[ \exists x \in D \mid x \text{ is not a perfect square} \]

Alternately, “Some multiples of 3 are not perfect squares.”
General forms for universal and existential quantification:

\[ \forall x \in X, \ P(x) \quad \exists x \in X \mid P(x) \]

\[ \forall x \in \emptyset, \ P(x) \text{ is always (vacuously) true.} \]

\[ \exists x \in \emptyset \mid P(x) \text{ is always false} \]
\[\sim (\forall x \in X, P(x))\]

\[\equiv \sim (P(x_1) \land P(x_2) \land \cdots)\]

\[\equiv \sim P(x_1) \lor \sim P(x_2) \lor \cdots \quad \text{By DeMorgan's Law}\]

\[\equiv \exists x \in X \mid \sim P(x)\]
1. Bob passed through $P$.
2. Bob passed through $N$.
3. Bob passed through $M$.
4. If Bob passed through $O$, then Bob passed through $F$.
5. If Bob passed through $K$, then Bob passed through $L$.
6. If Bob passed through $L$, then Bob passed through $K$.

Based on example by Susanna Epp, 2006
Let $X$ be the routes through the maze, that is, $X = \{CBGFONQR, CDILMNQR, CDIJKLMNQR\}$.

Let $P(x) = \text{route } x \text{ contains } L,$
$Q(x) = \text{route } x \text{ contains } K.$

Consider $\forall x \in X, P(x) \rightarrow Q(x)$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$P(x)$</th>
<th>$Q(x)$</th>
<th>$P(x) \rightarrow Q(x)$</th>
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</thead>
<tbody>
<tr>
<td>CBGFONQR</td>
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<tr>
<td>CDILMNQR</td>
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For next time:

Pg 133: 3.12.(1 & 2)
Pg 135: 3.13.(4 & 5)

Read 3.14