Chapter 3:

- Propositions, booleans, logical equivalence. §3.(1–4) (Today)
- Conditional propositions, conditional expressions. §3.(5–7) (Wednesday)
- Arguments. §3.(8 & 9) (Friday)
- Predicates and quantification. §3.(10–13) (Next week Monday)
- Quantified arguments. §3.14 (Next week Wednesday)

Today:

- Highlight main points of §3.(1&2): Propositions, forms, etc
- Demo SML features from §3.3: Boolean values
- Work through §3.4: Logical equivalences (Game 1)
Semester roadmap:

Ch 1 & 2: Raw materials
Ch 3: Formal logic
— Test 1, Sept 27 —
Ch 4: Proofs
Ch 5: Relations
— Test 2, Oct 29 —
Ch 6: Self reference
Ch 7: Functions
— Test 3, Dec 1 —

Chapter 3 roadmap:

Today: Logical equivalences (Game 1)
Wednesday: Conditionals (SML)
Friday: Arguments (Game 2)
Next week Monday: Predicates and quantification (SML)
Next week Wednesday: Quantified arguments (Game 3)
Next week Friday: Review for test
A **proposition** is a sentence that is true or false, but not both.

*It is snowing and it is not Thursday.*

A **propositional form** is like a proposition but with content replaced by variables.

* p and not q

\[ p \land \sim q \]
\( \mathbb{Z} = \{ \ldots -3, -2, -1, 0, 1, 2, 3 \ldots \} \)

\( \mathbb{B} = \{ T, F \} \)

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Evaluate (to $T$ or $F$) this logical expression:

$$(T \land (\sim F \lor F)) \land (T \land T)$$
Evaluate (to $T$ or $F$) this logical expression:

$$(T \lor F) \land \sim (F \land T)$$
Evaluate (to $T$ or $F$) this logical expression:

$$(F \lor F \lor T) \land (\sim T \land F)$$
\[ p \quad q \quad \sim p \quad \sim q \quad p \land q \quad \sim (p \land q) \quad \sim p \lor \sim q \]

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<td><strong>Commutative laws:</strong></td>
<td>( p \land q \equiv q \land p )</td>
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<td><strong>Associative laws:</strong></td>
<td>((p \land q) \land r \equiv p \land (q \land r))</td>
<td>((p \lor q) \lor r \equiv p \lor (q \lor r))</td>
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<td><strong>Distributive laws:</strong></td>
<td>( p \land (q \lor r) \equiv (p \land q) \lor (p \land r) )</td>
<td>( p \lor (q \land r) \equiv (p \lor q) \land (p \lor r) )</td>
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<td><strong>Absorption laws:</strong></td>
<td>( p \land (p \lor q) \equiv p )</td>
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<td><strong>Idempotent laws:</strong></td>
<td>( p \land p \equiv p )</td>
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<td><strong>Double negative law:</strong></td>
<td>( \sim \sim p \equiv p )</td>
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<td><strong>DeMorgan’s laws:</strong></td>
<td>( \sim (p \land q) \equiv \sim p \lor \sim q )</td>
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<td><strong>Negation laws:</strong></td>
<td>( p \lor \sim p \equiv T )</td>
<td>( p \land \sim p \equiv F )</td>
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<td><strong>Universal bound laws:</strong></td>
<td>( p \lor T \equiv T )</td>
<td>( p \land F \equiv F )</td>
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<td><strong>Identity laws:</strong></td>
<td>( p \land T \equiv p )</td>
<td>( p \lor F \equiv p )</td>
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<td><strong>Tautology and contradiction laws:</strong></td>
<td>( \sim T \equiv F )</td>
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Remember from high school algebra that there are “simplify” problems and “solve” problems.

■ Simplify \(3x(2 + 3x)^2 + 1\).

\[
3x(2 + 3x)^2 + 1 \\
= 3x(4 + 12x + 9x^2) + 1 \\
= 12x + 36x^2 + 27x^3 + 1 \\
= 27x^3 + 36x^2 + 12x + 1
\]

■ Solve \(12x = 57 - 7x\) for \(x\).

\[
12x = 57 - 7x \\
19x = 57 \\
x = 3
\]
Suppose we were to show that \( \sim (\sim p \land q) \lor (p \lor \sim p) \equiv p \lor \sim q \).

Do this:

\[
\begin{align*}
\sim (\sim p \land q) \lor (p \land \sim p) \\
\equiv \sim (\sim p \land q) \lor F \\
\equiv \sim (\sim p \land q) \\
\equiv p \lor \sim q
\end{align*}
\]

by negation law

by identity law

by De Morgan’s

Don’t do this:

\[
\begin{align*}
\sim (\sim p \land q) \lor (p \land \sim p) & \equiv p \lor \sim q \\
\sim (\sim p \land q) \lor F & \equiv p \lor \sim q \\
\sim (\sim p \land q) & \equiv p \lor \sim q \\
p \lor \sim q & \equiv p \lor \sim q
\end{align*}
\]

by negation law

by identity law

by De Morgan’s
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For next time:

Pg 102: 3.3.(5 & 6)
Pg 105: 3.4.(2, 4, 8-12)

Read 3.(5-7)
Take quiz