Chapter 3:

- Propositions, booleans, logical equivalence. §3.(1–4) (Today)
- Conditional propositions, conditional expressions. §3.(5–7) (Wednesday)
- Arguments. §3.(8 & 9) (Friday)
- Predicates and quantification. §3.(10–13) (next week Monday)
- Quantified arguments. §3.14 (next week Wednesday)

Today:

- Highlight main points of §3.(1&2): Propositions, forms, etc
- Demo SML features from §3.3: Boolean values
- Work through §3.4: Logical equivalences (Game 1)
Which phrase gives the best metaphor for the meaning of “set of sets”?

- Champion of champions
- Horror of horrors
- Box of boxes
- Friend of a friend

What is the cardinality of $\mathcal{P}(\emptyset)$?

If set $X$ has cardinality $n$, then what is the cardinality of $\mathcal{P}(X)$?
A **proposition** is a sentence that is true or false, but not both.

*It is snowing and it is not Thursday.*

A **propositional form** is like a proposition but with content replaced by variables.

\[ p \text{ and not } q \]

\[ p \land \sim q \]
\[\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}\]

\[\mathbb{B} = \{T, F\}\]

\[\begin{array}{c|cccc}
\times & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 2 & 3 \\
2 & 0 & 2 & 4 & 6 \\
3 & 0 & 3 & 6 & 9 \\
\end{array}\]

\[\begin{array}{c|cc}
\land & T & F \\
\hline
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F & F & F \\
\end{array}\]
\[\begin{array}{c|cc}
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T & T & F \\
F & F & F \\
\end{array}\]

\[\begin{array}{c|cc}
\lor & T & F \\
T & T & T \\
F & T & F \\
\end{array}\]

\[
\begin{array}{c|cc}
p & \sim p \\
T & F \\
F & T \\
\end{array}
\]

\[
\begin{array}{c|cc|c}
p & q & p \land q \\
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\]

\[
\begin{array}{c|cc|c}
p & q & p \lor q \\
T & T & T \\
T & F & T \\
F & T & T \\
F & F & F \\
\end{array}
\]
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<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land q$</th>
<th>$p \lor q$</th>
<th>$\sim p$</th>
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Evaluate (to $T$ or $F$) this logical expression:

$$(T \land (\sim F \lor F)) \land (T \land T)$$
Evaluate (to $T$ or $F$) this logical expression:

$$(T \lor F) \land \sim (F \land T)$$
Evaluate (to $T$ or $F$) this logical expression:

$$(F \lor F \lor T) \land (\sim T \land F)$$
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<th>$p$</th>
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<th>$\sim p$</th>
<th>$\sim q$</th>
<th>$p \land q$</th>
<th>$\sim (p \land q)$</th>
<th>$\sim p \lor \sim q$</th>
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<td>Laws</td>
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<td>Commutative laws:</td>
<td>$p \land q \equiv q \land p$</td>
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<td>$p \lor q \equiv q \lor p$</td>
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<td>Associative laws:</td>
<td>$(p \land q) \land r \equiv p \land (q \land r)$</td>
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<td>$(p \lor q) \lor r \equiv p \lor (q \lor r)$</td>
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<td>Distributive laws:</td>
<td>$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$</td>
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<td>$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$</td>
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<td>Absorption laws:</td>
<td>$p \land (p \lor q) \equiv p$</td>
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<td>$p \lor (p \land q) \equiv p$</td>
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<td>Idempotent laws:</td>
<td>$p \land p \equiv p$</td>
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<td>$p \lor p \equiv p$</td>
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<td>Double negative law:</td>
<td>$\sim \sim p \equiv p$</td>
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<td>DeMorgan’s laws:</td>
<td>$\sim (p \land q) \equiv \sim p \lor \sim q$</td>
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<td>$\sim (p \lor q) \equiv \sim p \land \sim q$</td>
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<td>Negation laws:</td>
<td>$p \lor \sim p \equiv T$</td>
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<td>$p \land \sim p \equiv F$</td>
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<td>Universal bound laws:</td>
<td>$p \lor T \equiv T$</td>
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<td>$p \land F \equiv F$</td>
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<td>Identity laws:</td>
<td>$p \land T \equiv p$</td>
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<td>$p \lor F \equiv p$</td>
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<td>Tautology and contradiction laws:</td>
<td>$\sim T \equiv F$</td>
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Remember from high school algebra that there are “simplify” problems and “solve” problems.

■ Simplify $3x(2 + 3x)^2 + 1$.

\[
3x(2 + 3x)^2 + 1 = 3x(4 + 12x + 9x^2) + 1 = 12x + 36x^2 + 27x^3 + 1 = 27x^3 + 36x^2 + 12x + 1
\]

■ Solve $12x = 57 - 7x$ for $x$.

\[
12x = 57 - 7x \\
19x = 57 \\
x = 3
\]
Suppose we were to show that $\sim (\sim p \land q) \lor (p \lor \sim p) \equiv p \lor \sim q$.

Do this:

\[
\begin{align*}
\sim (\sim p \land q) \lor (p \lor \sim p) & \\
\equiv \sim (\sim p \land q) \lor F & \text{by negation law} \\
\equiv \sim (\sim p \land q) & \text{by identity law} \\
\equiv p \lor \sim q & \text{by De Morgan’s}
\end{align*}
\]

Don’t do this:

\[
\begin{align*}
\sim (\sim p \land q) \lor (p \lor \sim p) & \equiv p \lor \sim q \\
\sim (\sim p \land q) \lor F & \equiv p \lor \sim q & \text{by negation law} \\
\sim (\sim p \land q) & \equiv p \lor \sim q & \text{by identity law} \\
p \lor \sim q & \equiv p \lor \sim q & \text{by De Morgan’s}
\end{align*}
\]
Semester roadmap:

Ch 1 & 2: Raw materials
Ch 3: Formal logic
— Test 1, Sept 25 —
Ch 4: Proofs
Ch 5: Relations
— Test 2, Oct 27 —
Ch 6: Self reference
Ch 7: Functions
— Test 3, Nov 29 —

Chapter 3 roadmap:

Today: Logical equivalences (Game 1)
Wednesday: Conditionals (SML)
Friday: Arguments (Game 2)
Next week Monday: Predicates and quantification (SML)
Next week Wednesday: Quantified arguments (Game 3)
Next week Friday: Review for test
For next time:

Pg 102: 3.3.(5 & 6)
Pg 105: 3.4.(2, 4, 8-12)
(See Canvas for a note about 3.4.(2 & 4))

Read 3.(5-7)
Take quiz