Universal instantiation
\[ \forall x \in A, \ P(x) \]
\[ a \in A \]
\[ \therefore P(a) \]

Universal modus ponens
\[ \forall x \in A, \ P(x) \rightarrow Q(x) \]
\[ a \in A \]
\[ P(a) \]
\[ \therefore Q(a) \]

Universal generalization
\[ \text{Suppose } a \in A \]
\[ P(a) \]
\[ \therefore \forall x \in A, \ P(x) \]

Hypothetical division
\[ \text{Suppose } p \]
\[ r \]
\[ \therefore r \]

Universal modus tollens
\[ \forall x \in A, \ P(x) \rightarrow Q(x) \]
\[ a \in A \]
\[ \sim Q(a) \]
\[ \therefore \sim P(a) \]

Existential Generalization
\[ a \in A \]
\[ P(a) \]
\[ \therefore \exists x \in A \mid P(x) \]

Hypothetical conditional
\[ \text{Suppose } p \]
\[ q \]
\[ \therefore p \rightarrow q \]

Existential instantiation
\[ \exists x \in A \mid P(x) \]
Let \( a \in A \mid P(a) \)
\[ \therefore a \in A \land P(a) \]
(Extra # 2)

(a) \( \forall x \in A, P(x) \)
(b) \( \forall x \in A, x \in B \lor R(x) \)
(c) \( \forall y \in B, Q(y) \lor \neg P(y) \)
(d) \( \forall x \in A, R(x) \rightarrow Q(x) \)
(e) \( \therefore \forall x \in A, Q(x) \)

Suppose \( a \in A \)

(i) \( a \in B \land R(a) \)
(ii) \( Q(a) \lor \neg P(a) \)
(iii) \( P(a) \)
(iv) \( Q(a) \)

Suppose \( a \in B \)

(v) \( Q(a) \)

Suppose \( R(a) \)

(vi) \( Q(a) \)

(iii) \( P(a) \)

(iv) \( Q(a) \)

(vi) \( Q(a) \)

(vii) \( \therefore \forall x \in A, Q(x) \)

by supposition, (b), and UI

by supposition, (c), and UI

by supposition, (a), and UI

by (ii), (iii), and elimination

by supposition, (c), and UMP

by (i), (iv),(v), and HDC

by supposition, (vi), and UG
(Extra # 3)

(a) \( \forall x \in A, P(x) \rightarrow R(x) \)
(b) \( \exists x \in A \mid P(x) \)
(c) \( \forall x \in A, Q(x) \lor x \in B \)
(d) \( \forall x \in A, P(x) \rightarrow \neg Q(x) \)
(e) \( \therefore \exists y \in B \mid R(y) \)

Let \( a \in A \mid P(a) \)

(i) \( a \in A \land P(a) \)
(ii) \( a \in A \)
(iii) \( P(a) \)
(iv) \( \neg Q(a) \)
(v) \( Q(a) \lor a \in B \)
(vi) \( a \in B \)
(vii) \( R(a) \)
(viii) \( \therefore \exists y \in B \mid R(y) \)

By (b) and EI
By (i) and specialization
By (i) and specialization
by (ii), (iii), (d), and UMP
by (ii), (c), and UI
by (iv), (v), and elimination
by (ii), (iii), (a), and UMP
by (vi), (vii), and EG
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Let \( a \in A \mid S(a) \)

(i) \( a \in A \land S(a) \) by (d) and El

(ii) \( a \in A \) by (i) and specialization

(iii) \( \exists y \in B \mid P(a, y) \) by (ii), (a), and UI

Let \( b \in B \mid P(a, b) \)

(iv) \( b \in B \land P(a, b) \) by (iii) and El

(v) \( b \in B \) by (iv) and specialization

(vi) \( P(a, b) \) by (iv) and specialization

(vii) \( \forall y \in B, P(a, y) \rightarrow \sim Q(y) \) by (c), (ii), UI

(viii) \( \sim Q(b) \) by (vii), (v), and UMP

(ix) \( Q(b) \lor R(b) \) by (b), (v), and UI

(x) \( R(b) \) by (ix), (vii), and elimination

(xi) \( \therefore \exists y \in B \mid R(y) \) by (v), (x), and EG
Suppose $a \in A$

1. $\forall x \in A, \ x \in B \land x \in C$
   (i) $a \in B \land a \in C$ by supposition, (a), and UI

2. $\forall x \in C, \ x \in D \lor x \in E$
   (ii) $a \in B$ by (i) and specialization
   (ii) $a \in C$ by (i) and specialization
   (iv) $a \in D \lor a \in E$ by (iii),(b), and UI

3. $\forall x \in B, \ x \in D \rightarrow P(x)$
   Suppose $a \in D$
   (v) $P(a)$ by (ii), supposition, (c), and UMP

4. $\forall x \in B, \ x \in E \rightarrow Q(x)$
   (vi) $P(a) \land Q(a)$ by (v) and (plain old) generalization

5. $\forall x \in B, \ (P(x) \lor Q(x)) \rightarrow R(x)$
   (vii) $Q(a)$ by (ii), supposition, (d), and UMP
   (viii) $P(a) \lor Q(a)$ by (vii) and (plain old) generalization
   (ix) $P(a) \lor Q(a)$ by (iv), (vi), (viii), and HDC
   (x) $R(a)$ by supposition, (ix), (e), and UMP
   (xi) $\therefore \forall x \in A, \ R(x)$ by supposition, (x), and UG.
Which of the following are true?

\[-((x - y) + (x - z)) = -(x - y) - (x - z)\]
\[-((x - y) + (x - z)) \cdot z = -(x - y) - (x - z) \cdot z\]
\[\sim (p \land q) \equiv \sim p \lor \sim q\]
\[\sim (p \land q) \land r \equiv \sim p \lor \sim q \land r\]
Which of the following are true?

\[(x + y) + z = x + (y + z)\]
\[(x - y) + z = x - (y + z)\]
\[(p \lor q) \lor r \equiv p \lor (q \lor r)\]
\[(p \lor q) \land r \equiv p \lor (q \land r)\]
1. Write a function `leastSigDigs` that takes a list of ints and returns a list of the least significant digits in those lists. For example, `leastSigDigs([283, 7234, 5, 2380])` would return `[3, 4, 5, 0]`.

2. Write a function `hasEmpty` that takes a list of lists (of any type) and determines whether or not the list of lists contains an empty list. For example, `hasEmpty([[1,2,3], [4,5], [], [6,7]])` would return `true`. 