<table>
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<tr>
<th><strong>Universal instantiation</strong></th>
<th><strong>Universal modus ponens</strong></th>
<th><strong>Universal generalization</strong></th>
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<tr>
<td>$\forall x \in A, \ P(x)$</td>
<td>$\forall x \in A, \ P(x) \rightarrow Q(x)$</td>
<td>Suppose $a \in A$</td>
</tr>
<tr>
<td>$a \in A$</td>
<td>$a \in A$</td>
<td>$P(a)$</td>
</tr>
<tr>
<td>$\therefore P(a)$</td>
<td>$P(a)$</td>
<td>$\therefore Q(a)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\therefore \forall x \in A, \ P(x)$</td>
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<tr>
<th><strong>Universal modus tollens</strong></th>
<th><strong>Existential Generalization</strong></th>
<th><strong>Hypothetical division</strong></th>
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<tr>
<td>$\forall x \in A, \ P(x) \rightarrow Q(x)$</td>
<td>$a \in A$</td>
<td>$p \lor q$</td>
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<td>$a \in A$</td>
<td>$P(a)$</td>
<td>Suppose $p$</td>
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<td>$\therefore Q(a)$</td>
<td>$P(a)$</td>
<td>$r$</td>
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<td>$\therefore \sim P(a)$</td>
<td>$\therefore \exists x \in A \mid P(x)$</td>
<td>Suppose $q$</td>
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<tr>
<td></td>
<td></td>
<td>$r$</td>
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<td></td>
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<td>$\therefore r$</td>
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<tr>
<th><strong>Existential instantiation</strong></th>
<th><strong>Hypothetical conditional</strong></th>
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<tr>
<td>$\exists x \in A \mid P(x)$</td>
<td>Suppose $p$</td>
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<tr>
<td>Let $a \in A \mid P(a)$</td>
<td>$q$</td>
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<td>$\therefore a \in A \land P(a)$</td>
<td>$\therefore p \rightarrow q$</td>
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</table>
(Extra # 2)

(a) \( \forall x \in A, P(x) \)
(b) \( \forall x \in A, x \in B \lor R(x) \)
(c) \( \forall y \in B, Q(y) \lor \sim P(y) \)
(d) \( \forall x \in A, R(x) \rightarrow Q(x) \)
(e) \( \therefore \forall x \in A, Q(x) \)

Suppose \( a \in A \)

(i) \( a \in B \land R(a) \)

Suppose \( a \in B \)

(ii) \( Q(a) \lor \sim P(a) \)

by supposition, (b), and UI

(iii) \( P(a) \)

by supposition, (a), and UI

(iv) \( Q(a) \)

by (ii), (iii), and elimination

Suppose \( R(a) \)

(v) \( Q(a) \)

by supposition, (c), and UMP

(vi) \( Q(a) \)

by (i), (iv),(v), and HDC

(vii) \( \therefore \forall x \in A, Q(x) \)

by supposition, (vi), and UG
(Extra # 3)

(a) \( \forall x \in A, P(x) \rightarrow R(x) \)

(b) \( \exists x \in A \mid P(x) \)

(c) \( \forall x \in A, Q(x) \vee x \in B \)

(d) \( \forall x \in A, P(x) \rightarrow \sim Q(x) \)

(e) \( \therefore \exists y \in B \mid R(y) \)

Let \( a \in A \mid P(a) \)

(i) \( a \in A \land P(a) \)

(ii) \( a \in A \)

(iii) \( P(a) \)

(iv) \( \sim Q(a) \)

(v) \( Q(a) \lor a \in B \)

(vi) \( a \in B \)

(vii) \( R(a) \)

(viii) \( \therefore \exists y \in B \mid R(y) \)

By (b) and EI

By (i) and specialization

By (i) and specialization

by (ii), (iii), (d), and UMP

by (ii), (c), and UI

by (iv), (v), and elimination

by (ii), (iii), (a), and UMP

by (vi), (vii), and EG
3.14.10

(a) \( \forall x \in A, \exists y \in B \mid P(x, y) \)
(b) \( \forall y \in B, Q(y) \lor R(y) \)
(c) \( \forall x \in A, y \in B, 
P(x, y) \rightarrow \sim Q(y) \)
(d) \( \exists x \in A \mid S(x) \)
(e) \( \therefore \exists y \in B \mid R(y) \)

Let \( a \in A \mid S(a) \)
(i) \( a \in A \land S(a) \) \hspace{1cm} by (d) and EI
(ii) \( a \in A \) \hspace{1cm} by (i) and specialization
(iii) \( \exists y \in B \mid P(a, y) \) \hspace{1cm} by (ii), (a), and UI

Let \( b \in B \mid P(a, b) \)
(iv) \( b \in B \land P(a, b) \) \hspace{1cm} by (iii) and EI
(v) \( b \in B \) \hspace{1cm} by (iv) and specialization
(vi) \( P(a, b) \) \hspace{1cm} by (iv) and specialization
(vii) \( \forall y \in B, P(a, y) \rightarrow \sim Q(y) \) \hspace{1cm} by (c), (ii), UI
(viii) \( \sim Q(b) \) \hspace{1cm} by (vii), (v), and UMP
(ix) \( Q(b) \lor R(b) \) \hspace{1cm} by (b), (v), and UI
(x) \( R(b) \) \hspace{1cm} by (ix), (vii), and elimination
(xi) \( \therefore \exists y \in B \mid R(y) \) \hspace{1cm} by (v), (x), and EG
3.14.11

(a) $\forall x \in A, \ x \in B \land x \in C$
(b) $\forall x \in C, \ x \in D \lor x \in E$
(c) $\forall x \in B, \ x \in D \rightarrow P(x)$
(d) $\forall x \in B, \ x \in E \rightarrow Q(x)$
(e) $\forall x \in B, \ (P(x) \lor Q(x)) \rightarrow R(x)$
(f) $\therefore \forall x \in A, \ R(x)$

Suppose $a \in A$

(i) $a \in B \land a \in C$ by supposition, (a), and UI
(ii) $a \in B$ by (i) and specialization
(iii) $a \in C$ by (i) and specialization
(iv) $a \in D \lor a \in E$ by (i) and specialization
(v) Suppose $a \in D$
(vi) $P(a)$ by (ii), supposition, (c), and UMP
(vii) $Q(a)$ by (ii), supposition, (d), and UMP
(viii) $P(a) \lor Q(a)$ by (vii) and (plain old) generalization
(ix) $P(a) \lor Q(a)$ by (iv), (vi), (viii), and HDC
(x) $R(a)$ by supposition, (ix), (e), and UMP
(xi) $\therefore \forall x \in A, \ R(x)$ by supposition, (x), and UG.
Which of the following are true?

\[ -((x - y) + (x - z)) = -(x - y) - (x - z) \]
\[ -((x - y) + (x - z)) \cdot z = -(x - y) - (x - z) \cdot z \]
\[ \sim (p \land q) \equiv \sim p \lor \sim q \]
\[ \sim (p \land q) \land r \equiv \sim p \lor \sim q \land r \]
Which of the following are true?

\[(x + y) + z = x + (y + z)\]
\[(x - y) + z = x - (y + z)\]
\[(p \lor q) \lor r \equiv p \lor (q \lor r)\]
\[(p \lor q) \land r \equiv p \lor (q \land r)\]
1. Write a function `leastSigDigs` that takes a list of ints and returns a list of the least significant digits in those lists. For example, `leastSigDigs[283, 7234, 5, 2380]` would return `[3, 4, 5, 0]`.

2. Write a function `hasEmpty` that takes a list of lists (of any type) and determines whether or not the list of lists contains an empty list. For example, `hasEmpty([[1,2,3], [4,5], [], [6,7]])` would return `true`. 