Semester roadmap:

Ch 1 & 2: Raw materials
Ch 3: Formal logic
— Test 1, Sept 27 —
Ch 4: Proofs
Ch 5: Relations
— Test 2, Oct 29 —
Ch 6: Self reference
Ch 7: Functions
— Test 3, Dec 1 —

Chapter 6 roadmap:

- Recursive definitions, recursive types (Today)
- Recursive proofs I: Structural induction (Friday)
- Recursive proofs II: Mathematical induction (next week Monday)
- Recursive proofs III: Loop invariants (next week Wednesday and Friday)
Axiom 7
There exists a whole number 0.

Axiom 8
Every whole number \( n \) has a successor, \( \text{succ } n \).

Axiom 9
No whole number has 0 as its successor.

Axiom 10
If \( a, b \in \mathbb{W} \), then \( a = b \) iff \( \text{succ } a = \text{succ } b \).

A whole number is either zero or one more than another whole number.

Compare to:
A list is either empty or an element together with its following list.
5 is a whole number because
5 is a whole number because it is the successor of 4, which is a whole number because
5 is a whole number because it is the successor of 4, which is a whole number because it is the successor of 3, which is a whole number because
5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because it is the successor of
2, which is a whole number because
5 is a whole number because it is the successor of 4, which is a whole number because it is the successor of 3, which is a whole number because it is the successor of 2, which is a whole number because it is the successor of 1, which is a whole number because
5 is a whole number because it is the successor of 4, which is a whole number because it is the successor of 3, which is a whole number because it is the successor of 2, which is a whole number because it is the successor of 1, which is a whole number because it is the successor of 0, which is a whole number by Axiom 7.
Lemmas for addition:

- $0 + b = b$
- $a + 0 = a$
- $a + b = (a + 1) + (b - 1)$

Lemmas for subtraction:

- $a - 0 = a$
- $a - b = (a - 1) - (b - 1)$

Lemmas for multiplication:

- $a \cdot 0 = 0$
- $0 \cdot b = 0$
- $a \cdot 1 = a$
- $a \cdot b = a + (a \cdot (b - 1))$
Tree

internal node

leaf

node
link
parent
child

root

internal node

leaf
Full Binary Tree
Expression trees:

datatype operation = Plus | Minus | Mul | Div;
datatype expression =  Internal of operation * expression * expression
  | Leaf of int;

$((5 - 7) \times ((3 + 2)/8))$

val exprExample = Internal(Mul, Internal(Minus,Leaf(5), Leaf(7)),
  Internal(Div,
    Internal(Plus, Leaf(3),
      Leaf(2)),
    Leaf(8)));

```
5
-    *
    \
7 +   /
     /
3   8
   / \
2
```
For next time:

Pg 260: 6.2.(6-8, 14-17)

Read 6.4