Semester roadmap:

Ch 1 & 2: Raw materials
Ch 3: Formal logic
— Test 1, Sept 25 —
Ch 4: Proofs
Ch 5: Relations
— Test 2, Oct 27 —
Ch 6: Self reference
Ch 7: Functions
— Test 3, Nov 29 —

Chapter 6 roadmap:

- Recursive definitions, recursive types (Today)
- Recursive proofs I: Structural induction (Wednesday)
- Recursive proofs II: Mathematical induction (Friday)
- Recursive proofs III: Loop invariants (next week Monday and Wednesday)

Project prototype due Wed, Nov 8
Axiom 7
There exists a whole number 0.

Axiom 8
Every whole number \( n \) has a successor, \( \text{succ} \ n \).

Axiom 9
No whole number has 0 as its successor.

Axiom 10
If \( a, b \in \mathbb{W} \), then \( a = b \) iff \( \text{succ} \ a = \text{succ} \ b \).

A whole number is either zero or one more than another whole number.

Compare to:
A list is either empty or an element together with its following list.
5 is a whole number because
5 is a whole number because it is the successor of 4, which is a whole number because
5 is a whole number because it is the successor of 4, which is a whole number because it is the successor of 3, which is a whole number because
5 is a whole number because it is the successor of 4, which is a whole number because it is the successor of 3, which is a whole number because it is the successor of 2, which is a whole number because
5 is a whole number because it is the successor of 4, which is a whole number because it is the successor of 3, which is a whole number because it is the successor of 2, which is a whole number because it is the successor of 1, which is a whole number because
5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because it is the successor of
2, which is a whole number because it is the successor of
1, which is a whole number because it is the successor of
0, which is a whole number by Axiom 7.
Lemmas for addition:

- $0 + b = b$
- $a + 0 = a$
- $a + b = (a + 1) + (b - 1)$

Lemmas for subtraction:

- $a - 0 = a$
- $a - b = (a - 1) - (b - 1)$

Lemmas for multiplication:

- $a \cdot 0 = 0$
- $0 \cdot b = 0$
- $a \cdot 1 = a$
- $a \cdot b = a + (a \cdot (b - 1))$
Tree

internal node

leaf

root

parent

color

node

link

child

internal node

leaf
Full Binary Tree
Expression trees:

datatype operation = Plus | Minus | Mul | Div;
datatype expression = Internal of operation * expression * expression
                   | Leaf of int;

\((5 - 7) \ast ((3 + 2)/8)\)

val exprExample = Internal(Mul, Internal(Minus,Leaf(5), Leaf(7)),
               Internal(Div,
                   Internal(Plus, Leaf(3),
                    Leaf(2)),
                   Leaf(8)));

\[
\begin{array}{c}
* \\
\downarrow \\
- \\
\downarrow \\
\downarrow \\
5 \\
\downarrow \\
+ \\
\downarrow \\
\downarrow \\
3 \\
\downarrow \\
\downarrow \\
7 \\
\downarrow \\
\downarrow \\
\downarrow \\
\downarrow \\
8 \\
\end{array}
\]
For next time:

Pg 260: 6.2.(6-8, 14-17)

Read 6.4
Take quiz