Chapter 1 outline:

- Introduction, sets and elements (this past Wednesday)
- Set operations; visual verification of set propositions (Today)
- Introduction to SML; cardinality and Cartesian products (Monday)
- Making types and functions in SML (Wednesday)
- More about functions in SML; introduction to lists [Chapter 2] (Next-week Friday)

Today:

- Set symbols and terminology
- Set notation
- Set operations
- Verifying set equivalence visually
5 is a natural number; or the collection of natural numbers contains 5. \(5 \in \mathbb{N}\)

Adding 0 to the collection of natural numbers makes the collection of whole numbers. \(\mathbb{N} \cup \{0\} = \mathbb{W}\)

Merging the algebraic numbers and the transcendental numbers makes the real numbers. \(\mathbb{A} \cup \mathbb{T} = \mathbb{R}\)

Transcendental numbers are those real numbers which are not algebraic numbers. \(\mathbb{T} = \mathbb{R} - \mathbb{A}\)

Nothing is both transcendental and algebraic, or the collection of things both transcendental and algebraic is empty. \(\mathbb{T} \cap \mathbb{A} = \emptyset\)

Negative integers are both negative and integers. \(\mathbb{Z}^- = \mathbb{Z} \cap \mathbb{R}^-\)

All integers are rational numbers. \(\mathbb{Z} \in \mathbb{R}\)

Since all rational numbers are algebraic numbers and all algebraic numbers are real numbers, it follows that all rational numbers are real numbers. \(\mathbb{Q} \subseteq \mathbb{A} \subseteq \mathbb{R} \therefore \mathbb{Q} \subseteq \mathbb{R}\)
Axiom (Existence.)

*There is a set with no elements.*

Axiom (Extensionality.)

*If every element of a set $X$ is an element of a set $Y$ and every element of $Y$ is an element of $X$, then $X = Y$.***
Union
\[ A \cup B = \{ x \mid x \in A \text{ or } x \in B \} \]
\[ \{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\} \]
\[ \{1, 2\} \cup \{3, 4\} = \{1, 2, 3, 4\} \]
\[ \{1, 2\} \cup \{1, 2, 3\} = \{1, 2, 3\} \]

Intersection
\[ A \cap B = \{ x \mid x \in A \text{ and } x \in B \} \]
\[ \{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\} \]
\[ \{1, 2\} \cap \{3, 4\} = \emptyset \]
\[ \{1, 2\} \cap \{1, 2, 3\} = \{1, 2\} \]

Difference
\[ A - B = \{ x \mid x \in A \text{ and } x \notin B \} \]
\[ \{1, 2, 3\} - \{2, 3, 4\} = \{1\} \]
\[ \{1, 2\} - \{3, 4\} = \{1, 2\} \]
\[ \{1, 2\} - \{1, 2, 3\} = \emptyset \]
1. \( \{1, 2, 3, 4, 5\} \cup \{5, 6, 7\} = \)

2. \( \{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\} = \)

3. \( \{1, 2, 3, 4, 5\} - \{2, 3, 4\} = \)

4. \( \{1, 2, 3, 4, 5\} - \{3, 4, 5, 6, 7\} = \)
1.4.7. $(A \cap B) - A$
1.4.8. \((A - B) \cup (B - A)\)
1.4.9. \((A \cup B) \cap (A \cup C)\)
1.4.10. \( (A \cap B) \cap (A \cup C) \)
(Exercises 1.4.(11–18).)

\[-12 \in \mathbb{R}^-.
\]

\[\mathbb{Q} \cap \mathbb{T} = \emptyset.\]

\[A \subseteq \mathbb{C}.\]

\[\frac{1}{63} \in \mathbb{Q} - \mathbb{R}.\]

\[\mathbb{R} \subseteq \mathbb{C} \cap \mathbb{R}^-\]

\[\mathbb{Z} - \mathbb{R}^- = \mathbb{W}.\]

\[4 \in \mathbb{C}.\]

\[\mathbb{T} \cup \mathbb{Z} \subseteq A.\]
\[ A \cup (A \cap B) = A \]
\[ A \cup \overline{A} = \mathcal{U} \]
\[ A \cup (B \cup C) = (A \cup B) \cup C \]
\[ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \]
$$A \cap B = A - (A - B)$$
\[(A \cap C) - (C - B) = A \cap B \cap C\]
\[ A \cup (A - B) = A \]
\[(A \cup (B - C)) \cap \overline{B} = A - B\]
For next time:

Pg 12: 1.3.(11-14, 16)
Pg 16: 1.4.(1-6, 19)
Pg 20: 1.5.(8-11)

Remember, submit through Schoology (as a PDF).

Read 1.(6-9)

Take quiz