Chapter 1 outline:

- Introduction, sets and elements (this past Monday)
- Set operations; visual verification of set propositions (Today)
- Introduction to SML; cardinality and Cartesian products (Friday)
- Making types in SML (next week Wednesday)
- Making functions in SML (next week Friday)

Today:

- Set symbols and terminology
- Set notation
- Set operations
- Verifying set equivalence visually
5 is a natural number; or the collection of natural numbers contains 5. \(5 \in \mathbb{N}\)

Adding 0 to the collection of natural numbers makes the collection of whole numbers.

\[\mathbb{N} \cup \{0\} = \mathbb{W}\]

Merging the algebraic numbers and the transcendental numbers makes the real numbers.

\[\mathbb{A} \cup \mathbb{T} = \mathbb{R}\]

Transcendental numbers are those real numbers which are not algebraic numbers.

\[\mathbb{T} = \mathbb{R} - \mathbb{A}\]

Nothing is both transcendental and algebraic, or the collection of things both transcendental and algebraic is empty.

\[\mathbb{T} \cap \mathbb{A} = \emptyset\]

Negative integers are both negative and integers.

\[\mathbb{Z}^- = \mathbb{Z} \cap \mathbb{R}^-\]

All integers are rational numbers.

\[\mathbb{Z} \in \mathbb{R}\]

Since all rational numbers are algebraic numbers and all algebraic numbers are real numbers, it follows that all rational numbers are real numbers.

\[\therefore \mathbb{Q} \subseteq \mathbb{R}\]
Axiom (Existence.)
There is a set with no elements.

Axiom (Extensionality.)
If every element of a set $X$ is an element of a set $Y$
and every element of $Y$ is an element of $X$, then $X = Y$. 
**Union**

\[ A \cup B = \{ x \mid x \in A \text{ or } x \in B \} \]

- \( \{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\} \)
- \( \{1, 2\} \cup \{3, 4\} = \{1, 2, 3, 4\} \)
- \( \{1, 2\} \cup \{1, 2, 3\} = \{1, 2, 3\} \)

**Intersection**

\[ A \cap B = \{ x \mid x \in A \text{ and } x \in B \} \]

- \( \{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\} \)
- \( \{1, 2\} \cap \{3, 4\} = \emptyset \)
- \( \{1, 2\} \cap \{1, 2, 3\} = \{1, 2\} \)

**Difference**

\[ A - B = \{ x \mid x \in A \text{ and } x \notin B \} \]

- \( \{1, 2, 3\} - \{2, 3, 4\} = \{1\} \)
- \( \{1, 2\} - \{3, 4\} = \{1, 2\} \)
- \( \{1, 2\} - \{1, 2, 3\} = \emptyset \)

\[ A \cap B = \{ x \mid x \in A \text{ and } x \in B \} \]

- \( \{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\} \)
- \( \{1, 2\} \cap \{3, 4\} = \emptyset \)
- \( \{1, 2\} \cap \{1, 2, 3\} = \{1, 2\} \)
1. $\{1, 2, 3, 4, 5\} \cup \{5, 6, 7\} =$

2. $\{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\} =$

3. $\{1, 2, 3, 4, 5\} - \{2, 3, 4\} =$

4. $\{1, 2, 3, 4, 5\} - \{3, 4, 5, 6, 7\} =$
Which of the following are equal to \( \{1, 2, 3, 4\} \)?

- \( \{1, 2\} \cup \{3, 4\} \)
- \( \{1, 2, 3\} \cup \{4\} \)
- \( \{1, 2, 3\} \cup \{2, 3, 4\} \)
- \( \{1, 2, 3\} \cup \{3, 4, 5\} \)
- \( \{2, 3\} \cup \{1, 4\} \)
- \( \{1\} \cup \{3, 4\} \)
- \( \{4, 3, 2, 1\} \)
- \( \{1\} \cup \{1, 2\} \cup \{1, 2, 3\} \cup \{1, 2, 3, 4\} \)
1.4.7. \((A \cap B) - A\)
1.4.8. \((A - B) \cup (B - A)\)
1.4.9. \((A \cup B) \cap (A \cup C)\)
1.4.10. \((\overline{A \cap B}) \cap (A \cup C)\)
\[ A \cup (A \cap B) = A \]
\[ A \cup \overline{A} = \mathcal{U} \]
\[ A \cup (B \cup C) = (A \cup B) \cup C \]
\[ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \]
\[ A \cap B = A - (A - B) \]
\[(A \cap C) - (C - B) = A \cap B \cap C\]
\[ A \cup (A - B) = A \]
\[(A \cup (B - C)) \cap \overline{B} = A - B\]
For next time:

Pg 12: 1.3.(11-14, 16)
Pg 16: 1.4.(1-6, 19)
Pg 20: 1.5.(8-11)

Read 1.(6-9)

Take quiz