Transitions from discrete to continuous probability units

- Common discrete distributions (last week Friday and this week Monday)
- Introduction to continuous random variables (**Today**)
- Continuous distributions (Friday)

Today:

- Finishing the geometric distribution
- The intuition of continuous random variables
- Probability density functions
- Basic propositions about continuous random variables

Let $\chi = \{0, 1\}$, and let $p \in [0, 1]$. Then random variable X has a **Bernoulli distribution** if

$$p(x) = \begin{cases} p & \text{if } x = 1 \\ (1-p) & \text{otherwise} \end{cases}$$

$$= p^x (1-p)^{1-x}$$

Let $\chi = \mathbb{Z}_{n+1} = \{0, 1, 2, \ldots n\}$ for some $n \in \mathbb{N}$ and $p \in [0, 1]$. Then random variable X has a **binomial distribution** if

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Let $\chi = \mathbb{W}$ and $\lambda \in \mathbb{R}^+$. Then random variable X has a **Poisson distribution** if

$$p(x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

Let $\chi = \mathbb{N}$ and $p \in [0, 1]$. Then random variable X has a **geometric distribution** if

$$p(x) = (1-p)^{x-1} p$$

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An urn contains 10 white balls and 3 black balls. Balls are drawn one at a time, with replacement, until a black one is drawn.

1. What is the probability that exactly 4 draws are needed?
2. What is the probability that at least 6 draws are needed?

Adapted from Ross, *A First Course in Probability*, 1997 , pg 162.

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Theorem 1

If X is a geometric random variable with parameter p and probability mass function $p()$, then

a. $\sum_{x=1}^{\infty}(1-p)^{x-1}p = 1$

b. $E[X] = \frac{1}{p}$

c. $Var(X) = \frac{1-p}{p^2}$

A random variable $X : \Omega \to \chi$ is **continuous** if χ is an uncountable subset of $\mathbb{R}$ (or just $\chi = \mathbb{R}$).

Let $F : \chi \to [0, 1]$ be the cumulative distribution function for X. Then the **probability density function** for X is the derivative of F, that is,

$$f(x) = F'(x)$$

Moreover, if P is the probability function for sample space Ω, then for $a, b \in \chi$,

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

Properties of continuous random variables and their probability density functions

$$\forall\ x \in \chi, f(x) >= 0$$

$$P(-\infty \leq X \leq \infty) = \int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$P(X = a) = \int_{a}^{a} f(x)\, dx = 0$$

$$E[X] = \int_{-\infty}^{\infty} x\ f(x)\, dx$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)\ f(x)\, dx$$

$$E[aX + b] = aE[X] + b$$

For next time:

Take the quiz on Canvas