Continuous probability unit

- Introduction to continuous random variables (last weekWednesday)
- Continuous distributions 1: Uniform ~~and Gaussian~~ (last week Friday)
- Continuous distributions 2: Gaussian and Exponential ~~and Pareto~~ (**Today**)
- Jointly distributed random variables (Wednesday)

Today:

- The Gaussian (normal) distribution
- The exponential distribution

Let X be a continuous random variable and let $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$. Then X has a **normal** or **Gaussian** distribution if its probability density function f is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Moreover,

$$E[X] = \mu$$

and

$$Var(X) = \sigma^2$$

Students in evil Professor Samoht NenurdNav's course (CSCI 720 Tortuous Math Topics) are anxious after having taken a very challenging test. "Oh, don't worry, he *curves* the grades," one confident student says to cheer his classmates up.

Professor NenurdNav assumes that, with a sufficient sample size, test scores will be distributed on a bell curve. He computes μ and σ from the test scores and assigns As to scores greater than $\mu + \sigma$ (one standard deviation above average), Bs to scores between μ and $\mu + \sigma$ (within one standard deviation above average), Cs to scores between $\mu - \sigma$ and *mu* (within one standard deviation below average), Ds to scores between $\mu - 2\sigma$ and $\mu - \sigma$, and Fs to scores below $\mu - 2\sigma$.

What percentage of students are likely to get each grade?

Let X be a continuous random variable and let $\lambda \in \mathbb{R}^+$ Then X has an **exponential** distribution if its probability density function f is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

> *The exponential distribution often arises, in practice, as being the distribution of the amount of time until some specific event occurs.*
>
> *Ross, A First Course in Probability, 1997 , pg 217*

Compare with the Poisson distribution. Let $\chi = \mathbb{W}$ and $\lambda \in \mathbb{R}^+$. Then random variable X has a **Poisson distribution** if

$$p(x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

For next time:

Do the exercise at the bottom of the backside of the handout.