Chapter 5, Binary search trees:

- Binary search trees; the balanced BST problem (fall-break eve; finished last week Friday)
- AVL trees (last week Friday and this past Monday)
- Traditional red-black trees (last week Wednesday)
- Left-leaning red-black trees (Today)
- “Wrap-up” BSTs (next week Monday)
- Begin dynamic programming (next week Wednesday)

Today:

- LLRB context and definition
- LLRB invariant and cases
- Performance comparison among AVL, TrRB, and LLRB
Why invariants?

- An invariant is a constraint we put on our code to help us guarantee something about it.
- The general invariant for BSTs guarantees the correctness of our find algorithm.
- The invariants for AVL trees and RB trees guarantee logarithmic-time operations.

A stronger constraint is both a stronger constraint to maintain and a stronger constraint to assume.
A left-leaning red-black tree is a binary tree (usually a BST) that is either empty or it is rooted at node \( T \) such that

- \( T \) is either red or black.
- Both of \( T \)'s children are roots of left-leaning red-black trees.
- \( T \)'s right child is black.
- If \( T \) is red, then its left child is black.
- The left-leaning red-black trees rooted at its children have equal blackheight; moreover, the blackheight of the tree rooted at \( T \) is one more than the blackheight of its children if \( T \) is black or equal to that of its children if \( T \) is red.
The first came out red, all his body like a hairy cloak, so they called his name Esau. Gen 25:25

Yet I have loved Jacob, but Esau I have hated. Mal 1:2&3, qtd in Rom 9:13
Left-leaning

Traditional
Left-leaning

Traditional
Left-leaning

Traditional
Left-leaning

Traditional
Left-leaning

Traditional
Left-leaning

Traditional
Left-leaning

Traditional
Potential violations

Ignorant node

Inconsistent backheight

Red null

Double red

Right red

\{ shouldn't happen

\{ fix when they happen
Invariant 28 (Postconditions of RealNode.put() with LLRBBalancer.) Let \( x \) be the root of a subtree on which put() is called and let \( y \) be the node returned, that is, the root of the resulting subtree.

(a) The subtree rooted at \( y \) has a consistent black height.

(b) The black height of subtree rooted at \( y \) is equal to the original black height of the subtree rooted at \( x \).

(c) The subtree rooted at \( y \) has no double-red violations except, possibly, both \( y \) and its left child is red, which can happen only if \( x \) is a left child.

(d) The subtree rooted at \( y \) has no right-red violations.
redden B
blacken A and C
B gets A's color
redden A
α
rotate left about A
α
B
γβ
A
γ
B
β
rotate right about C
redden A
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