Chapter 5, Binary search trees:
  ▶ Binary search trees; the balanced BST problem (before break)
  ▶ AVL trees (last week Monday and Wednesday)
  ▶ Traditional red-black trees (last week Friday)
  ▶ Left-leaning red-black trees (Today)
  ▶ “Wrap-up” BSTs (Wednesday)
  ▶ Begin dynamic programming (Friday)

Today:
  ▶ LLRB context and definition
  ▶ LLRB invariant and cases
  ▶ Performance comparison among AVL, TrRB, and LLRB
Why invariants?

- An invariant is a constraint we put on our code to help us guarantee something about it.
- The general invariant for BSTs guarantees the correctness of our find algorithm.
- The invariants for AVL trees and RB trees guarantee logarithmic-time operations.

A stronger constraint is both a stronger constraint to maintain and a stronger constraint to assume.
A left-leaning red-black tree is a binary tree (usually a BST) that is either empty or it is rooted at node $T$ such that

- $T$ is either red or black.
- Both of $T$’s children are roots of left-leaning red-black trees.
- $T$’s right child is black.
- If $T$ is red, then its left child is black.
- The left-leaning red-black trees rooted at its children have equal blackheight; moreover, the blackheight of the tree rooted at $T$ is one more than the blackheight of its children if $T$ is black or equal to that of its children if $T$ is red.
The first came out red, all his body like a hairy cloak, so they called his name Esau.
Gen 25:25

Yet I have loved Jacob, but Esau I have hated.
Mal 1:2&3, qtd in Rom 9:13
Left-leaning

Traditional
Left-leaning

Traditional
Left-leaning

```
  D
 /\  \
B /  \
/    \
A    C
```

Traditional

```
  D
 /\  \
B /  \
/    \
A    C
```
Left-leaning

```
D
 /   /
B   E
|   |
A   C
```

Traditional

```
D
 /   /
B   E
|   |
A   C
```
Potential violations

- Ignorant node
- Inconsistent backheight
- Red null
- Double red
- Right red

\{ shouldn’t happen  \\
\{ fix when they happen  \\

Invariant 28 (Postconditions of RealNode.put() with LLRBBalancer.) Let $x$ be the root of a subtree on which put() is called and let $y$ be the node returned, that is, the root of the resulting subtree.

(a) The subtree rooted at $y$ has a consistent black height.
(b) The black height of subtree rooted at $y$ is equal to the original black height of the subtree rooted at $x$.
(c) The subtree rooted at $y$ has no double-red violations except, possibly, both $y$ and its left child is red, which can happen only if $x$ is a left child.
(d) The subtree rooted at $y$ has no right-red violations.
redden B
go to blacken A and C
B gets A's color
redden A
rotate left about A
B gets A's color
redden A
rotate right about C
redden A
<table>
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<th>Blackheight</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tr>
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<td>4</td>
<td>6</td>
<td>8</td>
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<tr>
<td>Nodes</td>
<td>2</td>
<td>6</td>
<td>14</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Height</td>
<td>Leaf %</td>
<td>Total depth</td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>--------</td>
<td>--------</td>
<td>-------------</td>
<td>----------------</td>
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<td><strong>After puts</strong></td>
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<td></td>
<td><strong>After removals</strong></td>
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<td><strong>Unbalanced</strong></td>
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<td>33.3%</td>
<td>134507</td>
<td>28</td>
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<tr>
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<td>31</td>
<td>33.2%</td>
<td>127865</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>33.1%</td>
<td>129037</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>33.5%</td>
<td>124463</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>32</td>
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<td>100395</td>
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<td>100282</td>
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Coming up:

* Do **BST rotations** project *(suggested by last week Wednesday, Mar 16)*
* Do **AVL** project *(suggested by today, Mar 21)*
* Do **Traditional RB** project *(suggested by Monday, Mar 28)*
* *(Recommended: Do **Left-leaning RB** project for your own practice)*

**Due Wed, Mar 23** *(end of day) (but spread it out)*
Read Sections 5.(4-6) *[some parts carefully, some parts skim, some parts optional—see Schoology]*
Do Exercise 5.14
Take quiz

**Due Fri, Mar 25** *(end of day)*
Read Section 6.(1&2)
Do Exercises 6.(5–7)
Take quiz